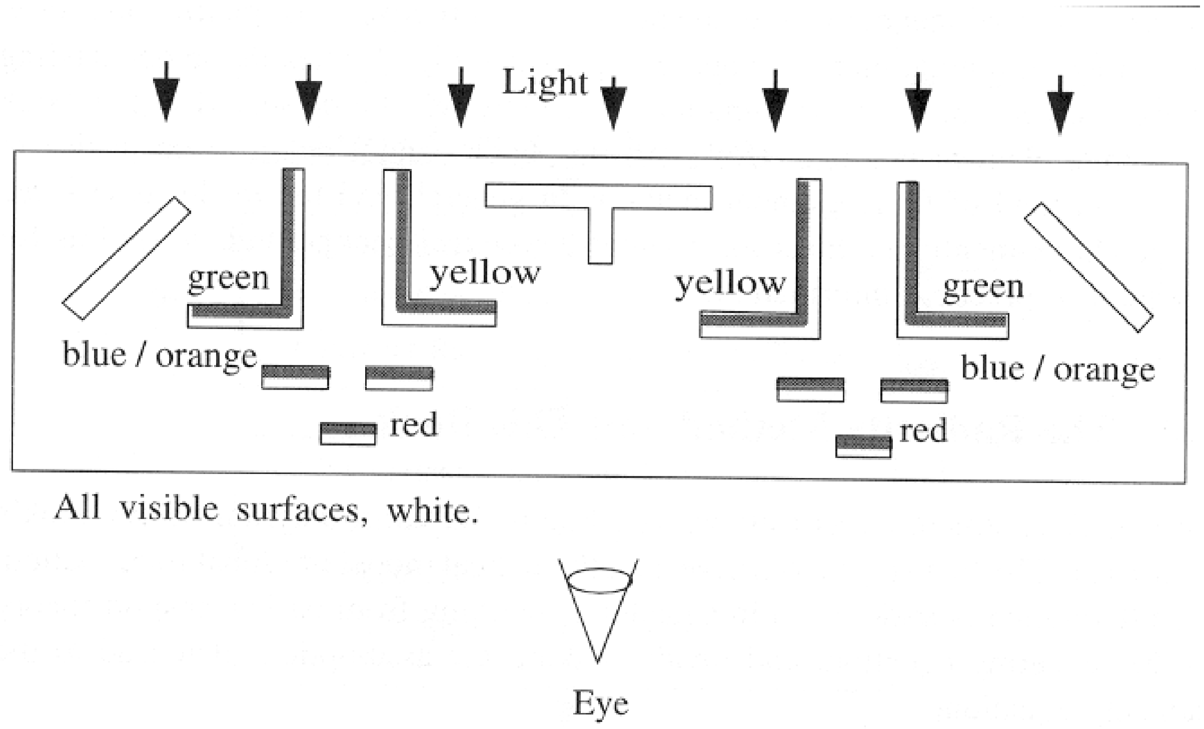
CS 428: Fall 2009

Introduction to Computer Graphics

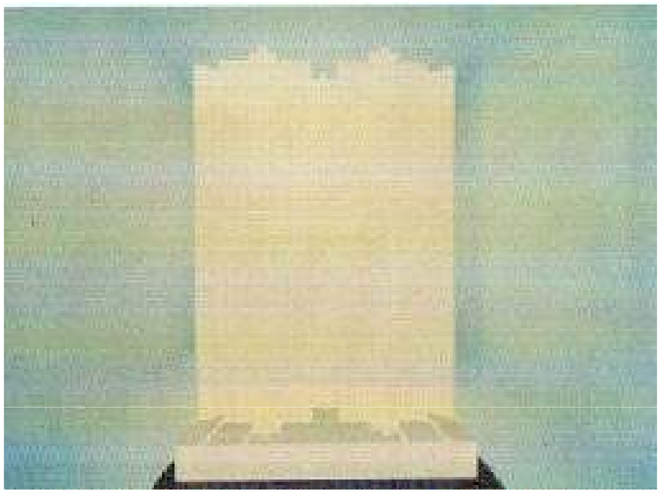
Radiosity

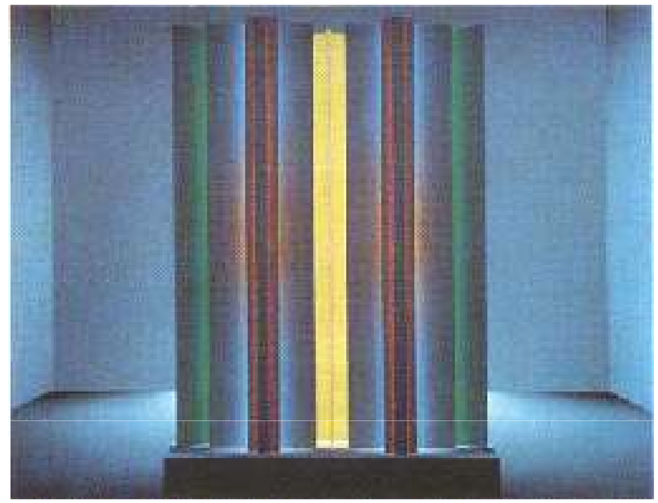
Problems with diffuse lighting

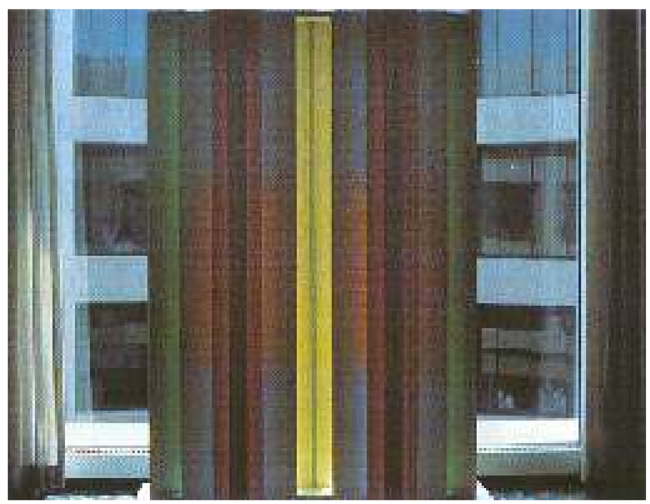


A Daylight Experiment, John Ferren

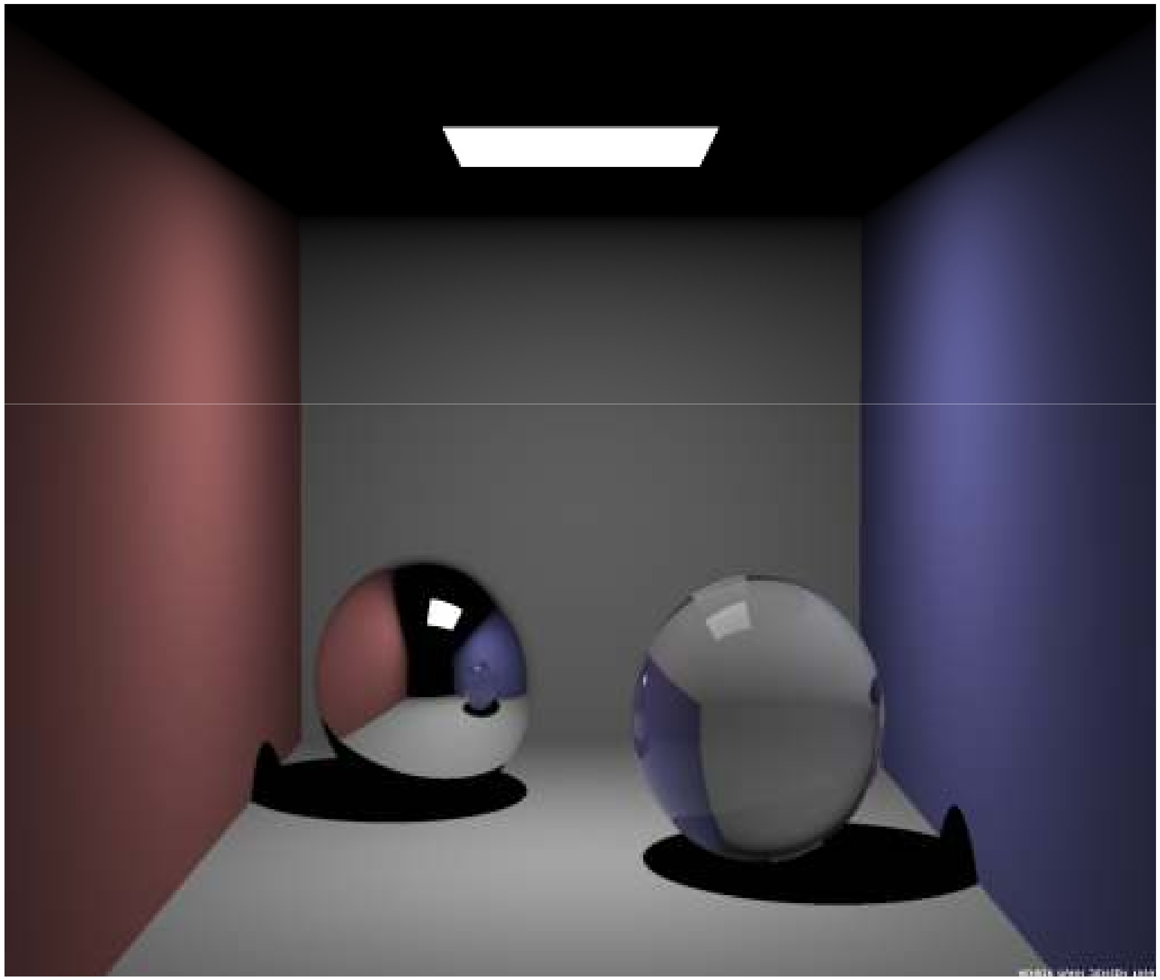
Problems with diffuse lighting



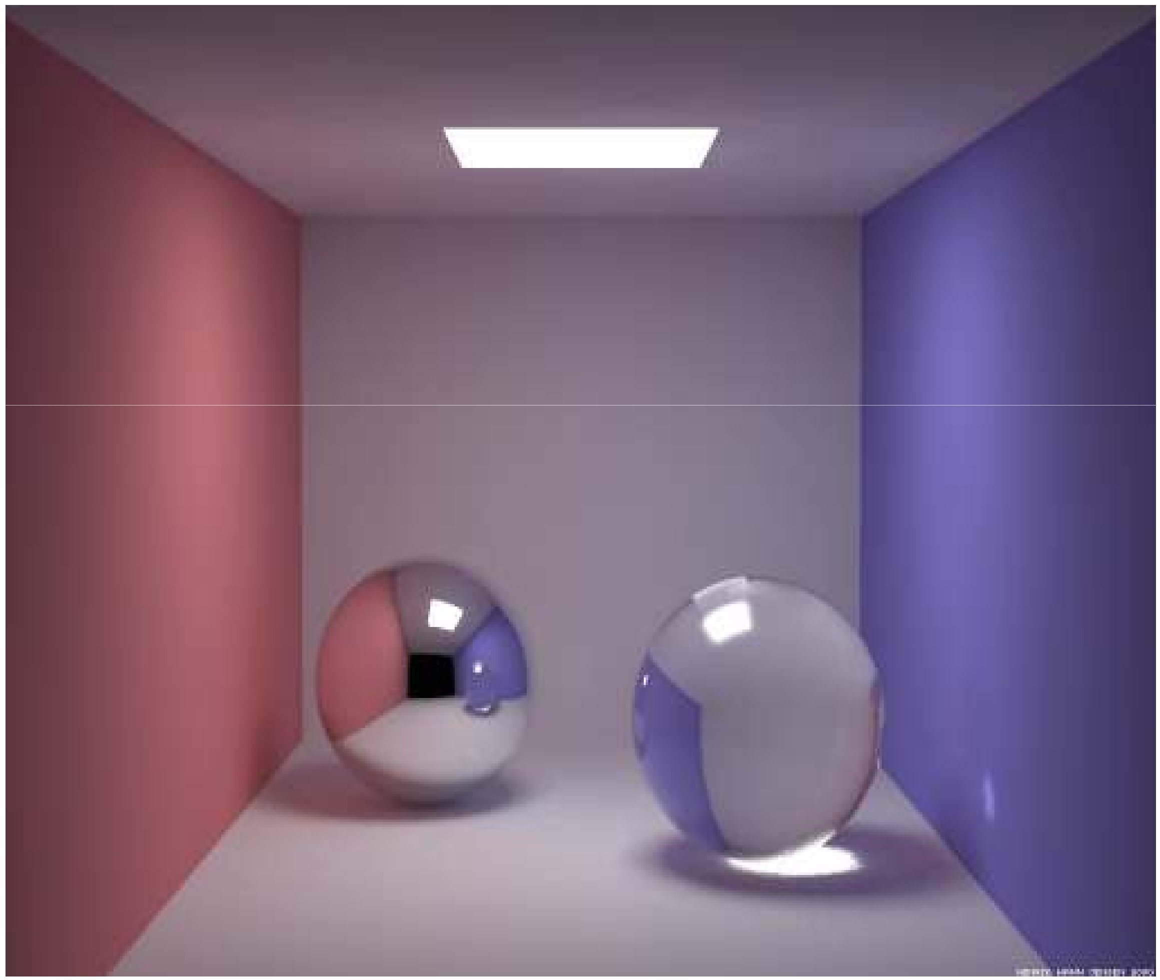




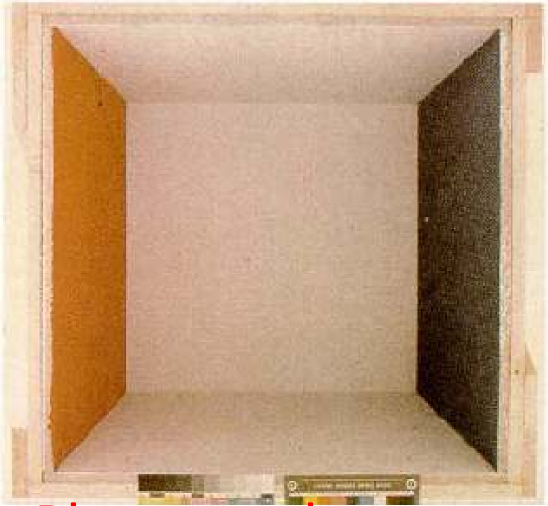
Direct lighting



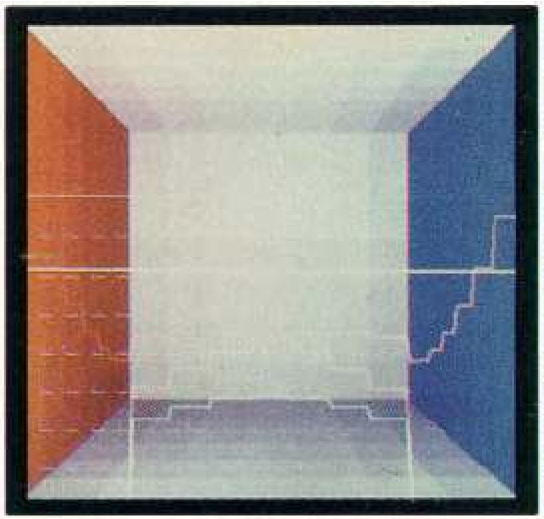
Global lighting



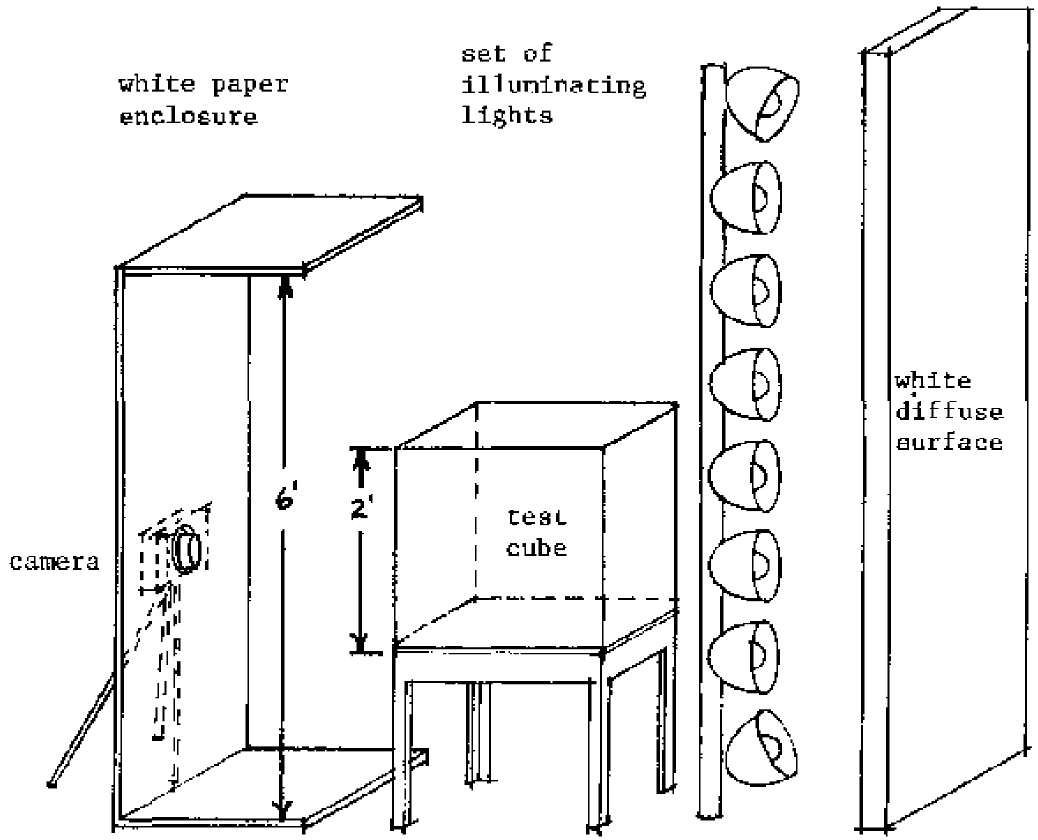
Cornell box



Photography



Simulation



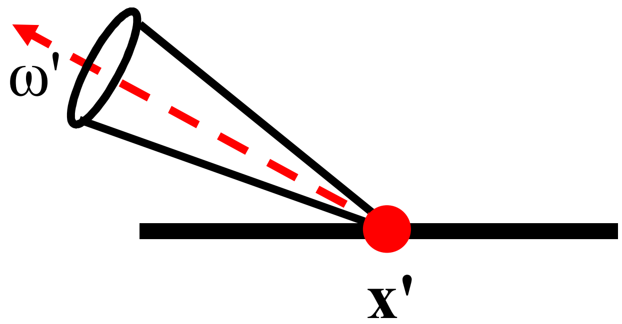
Goral, Torrance, Greenberg & Battaile
Modeling the Interaction of Light Between Diffuse Surfaces
SIGGRAPH '84

Cornell box

- Calibration and measurement allows comparisons between reality and simulation

Light Measurement Laboratory
Cornell University, Program for Computer Graphics

The rendering equation



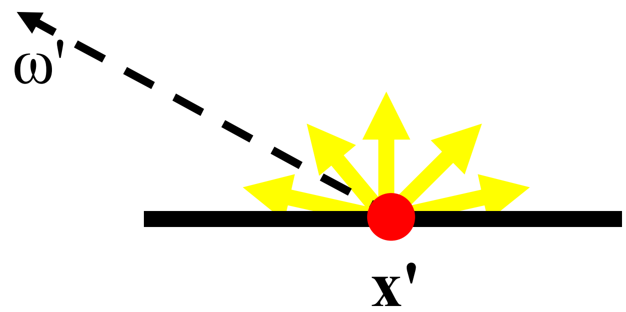
$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\, dA$$

$L(x',\omega')$ is the radiance from point x' in direction of ω'

Radiance is measured in $[W/(m^2 \cdot sr)]$

`http://en.wikipedia.org/wiki/Radiance`

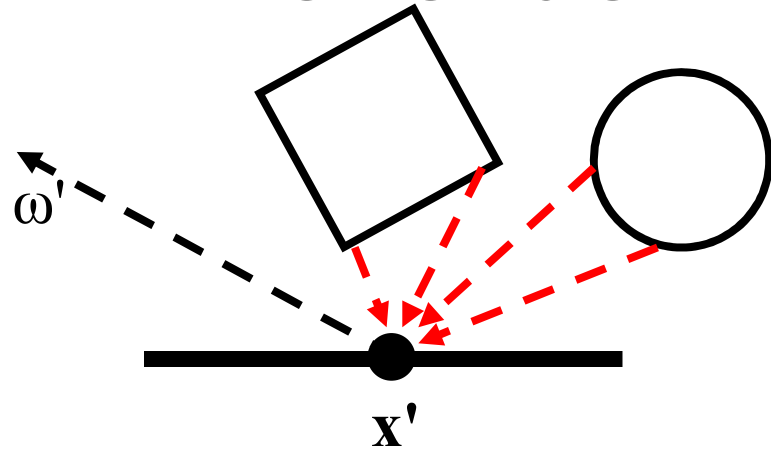
The rendering equation



$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\,dA$$

$E(x',\omega')$ is the emitted radiance: E is greater zero for light sources

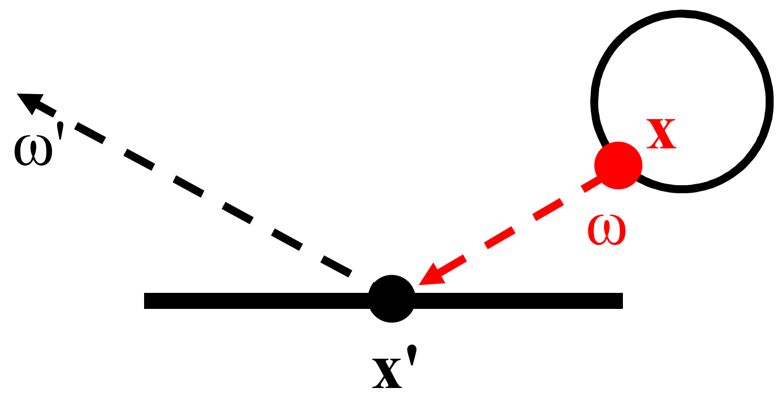
The rendering equation



$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\,dA$$

Sum of contributions from all other scene elements to the radiance from point x' in direction of ω'

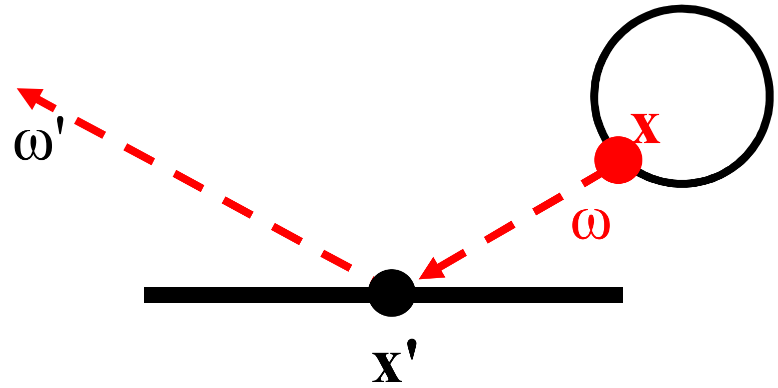
The rendering equation



$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\, dA$$

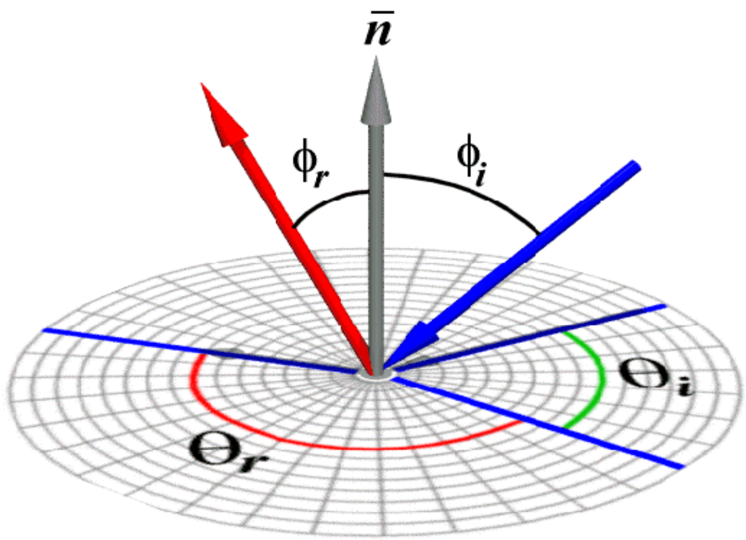
For every x, compute $L(x, \omega)$, the radiance in point x in direction ω (from x to x')

The rendering equation

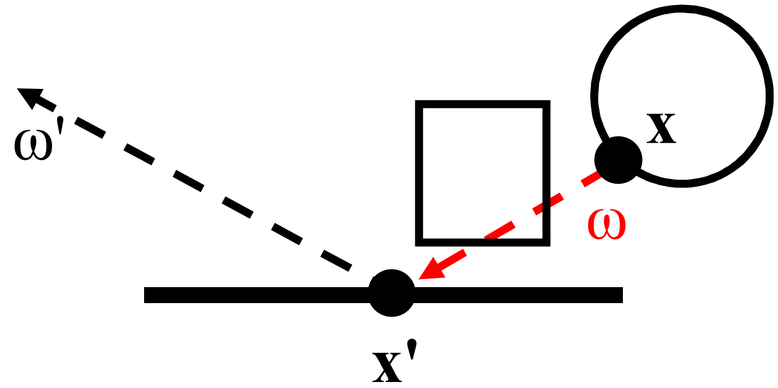


$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega') L(x,\omega) G(x,x') V(x,x')\, dA$$

The contribution is scaled by $\rho_{x'}(\omega,\omega')$ (the BRDF in x')



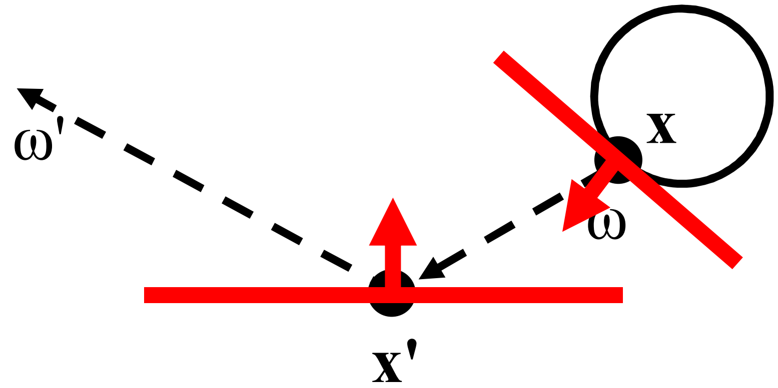
The rendering equation



$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\,dA$$

For every x, determine $V(x,x')$, the visibility from x relative to x': 1 if there is no occlusion in direction ω, 0 otherwise

The rendering equation

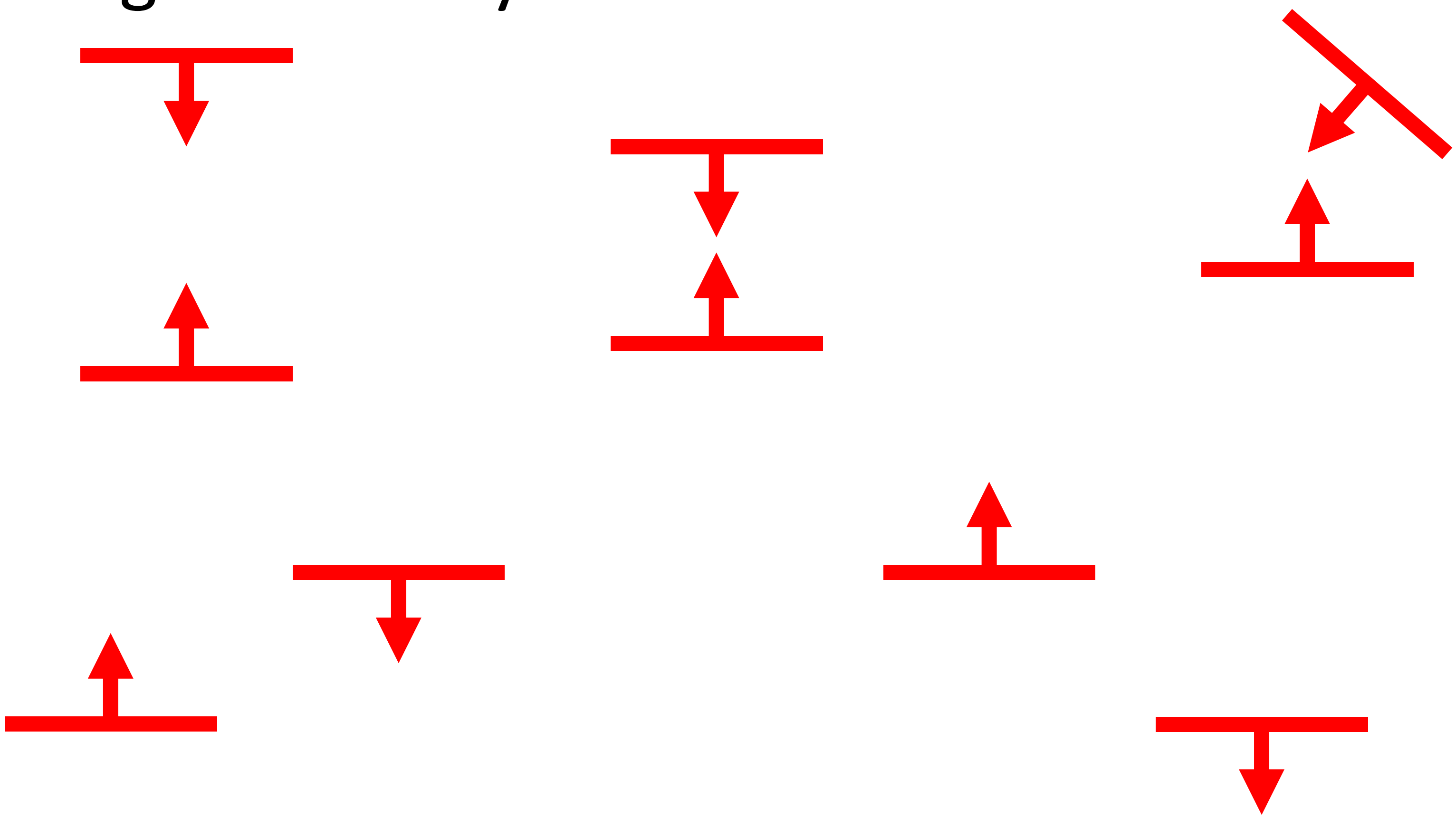


$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega') L(x,\omega) G(x,x') V(x,x')\, dA$$

For every x, compute $G(x, x')$, the geometry term w.r.t. x and x'

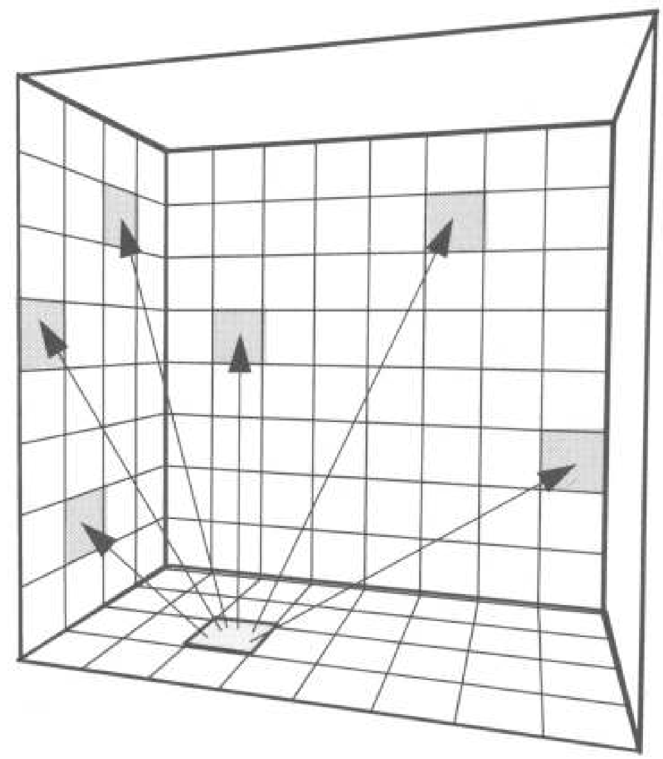
G(x,x')?

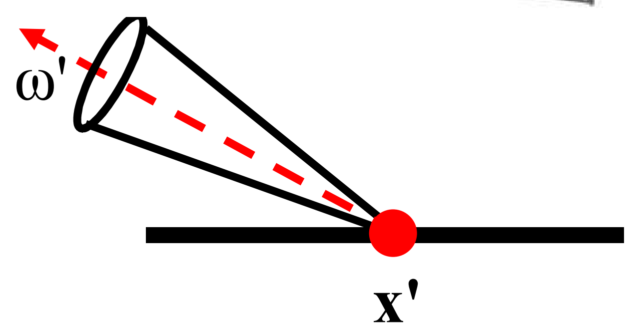
- Which constellation leads to a large exchange of light and why?



The radiosity assumptions

- Surfaces are Lambertian (perfectly diffuse)
  - Reflection occurs in all directions
- The scene is split into small surface elements
- The radiosity B_i, is the total radiosity that comes from element i
- For each element, the radiosity is constant



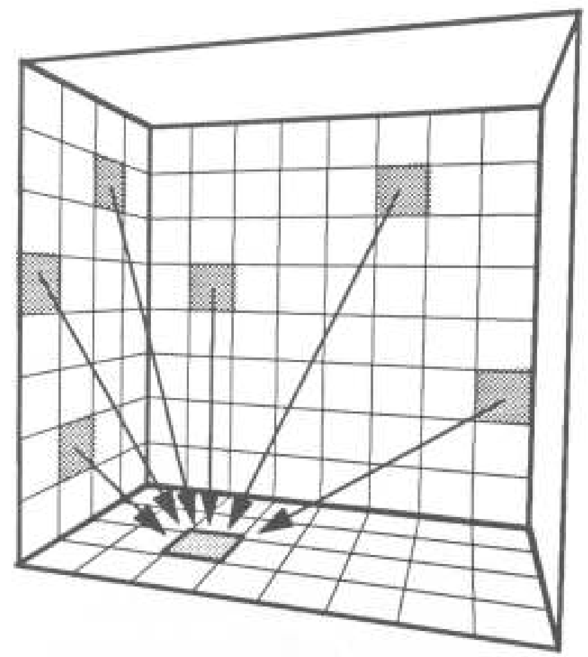


The radiosity equation

$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega') L(x,\omega) G(x,x') V(x,x')\, dA$$

Radiosity assumption:
Perfectly diffuse surfaces – no directional dependency

$$B_{x'} = E_{x'} + \rho_{x'} \int B_x \; G(x,x') V(x,x')$$



The radiosity equation

- Continuous radiosity equation

Reflection factor

$$B_{x'} = E_{x'} + \rho_{x'} \int G(x,x')\, V(x,x')\, B_x$$

Form factor

  - G: geometry term
  - V: visibility term
- Properties
  - No analytical solution, even for simple scenes

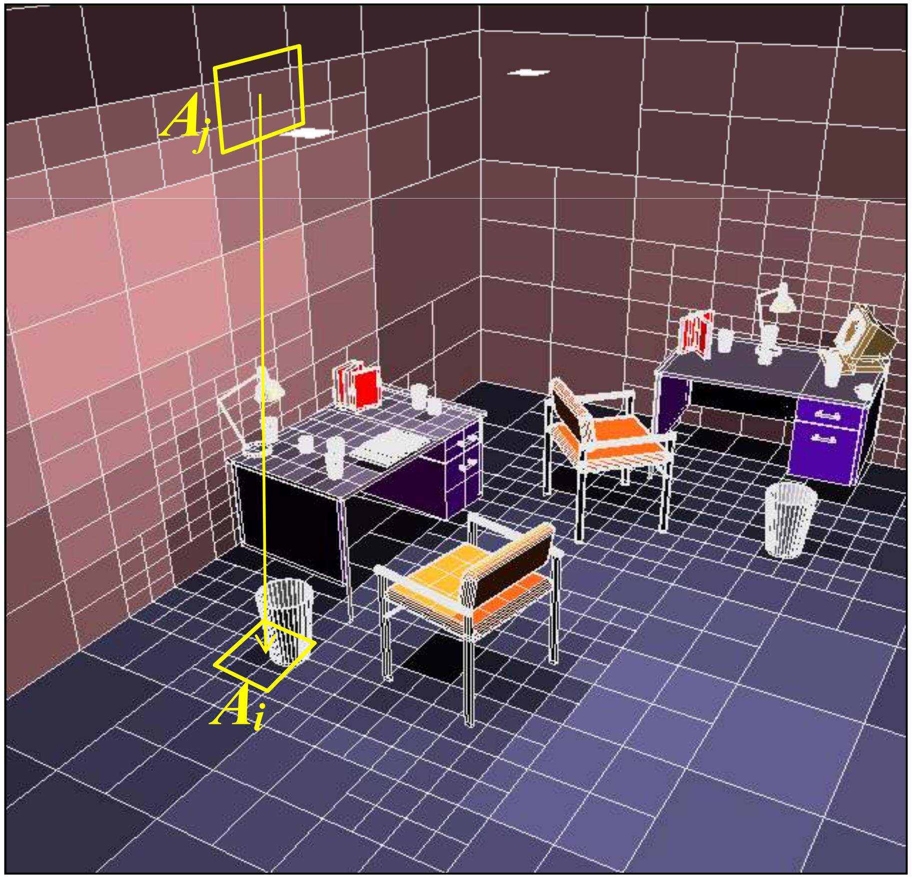
The radiosity equation

- Discretize into elements with const. radiosity

Reflection factor

$$B_i = E_i + \rho_i \sum_{j=1}^{n} F_{ij} B_j$$

Form factor



- Properties
  - Iterative solution
  - Expensive geometry computations

The radiosity matrix

$$B_i = E_i + \rho_i \sum_{j=1}^{n} F_{ij} B_j$$

- n linear equations in n unknowns B_i:

$$\begin{bmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & & \\ \vdots & & \ddots & \\ -\rho_n F_{n1} & \cdots & \cdots & 1-\rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

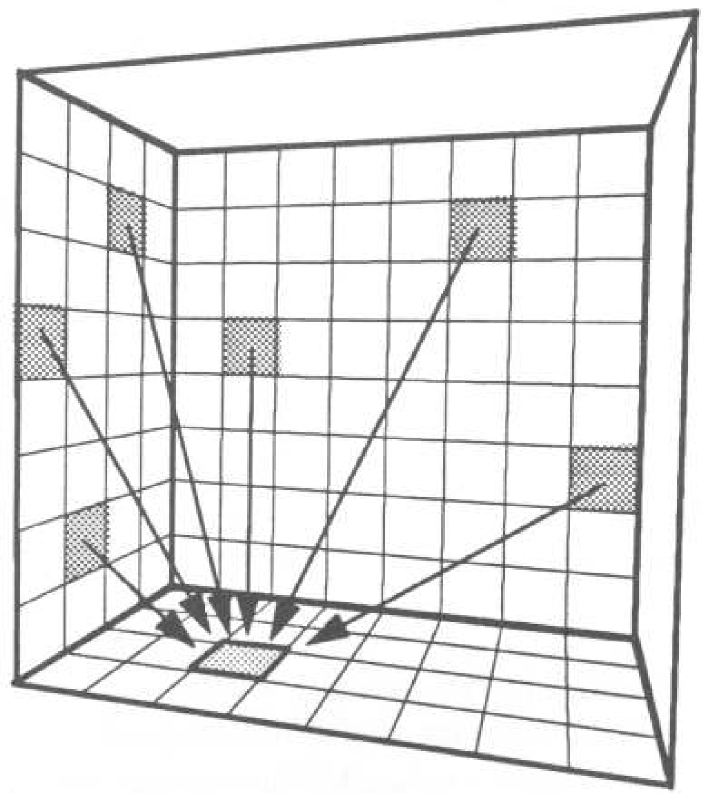
- The solution of this LSE results in B_i, which are independent of viewer position and direction

The radiosity matrix

Iterative solution

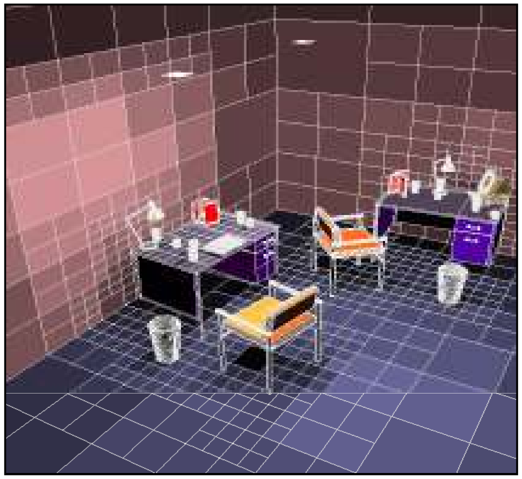
- The radiosity of an element is replaced by the multiplication of a row with the current solution vector (Gathering) (= Gauss-Seidel iteration)

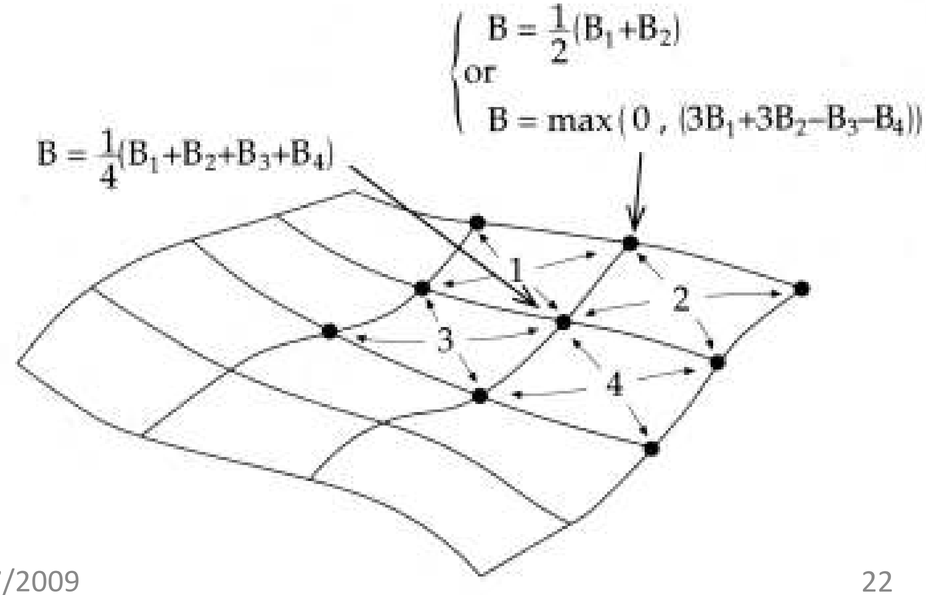
$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_i \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_i \\ \vdots \\ E_n \end{bmatrix} + \begin{bmatrix} & & & \\ & & & \\ \rho_i F_{i1} & \rho_i F_{i2} & \cdots & \rho_i F_{in} \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_i \\ \vdots \\ B_n \end{bmatrix}$$



Rendering the radiosity solution

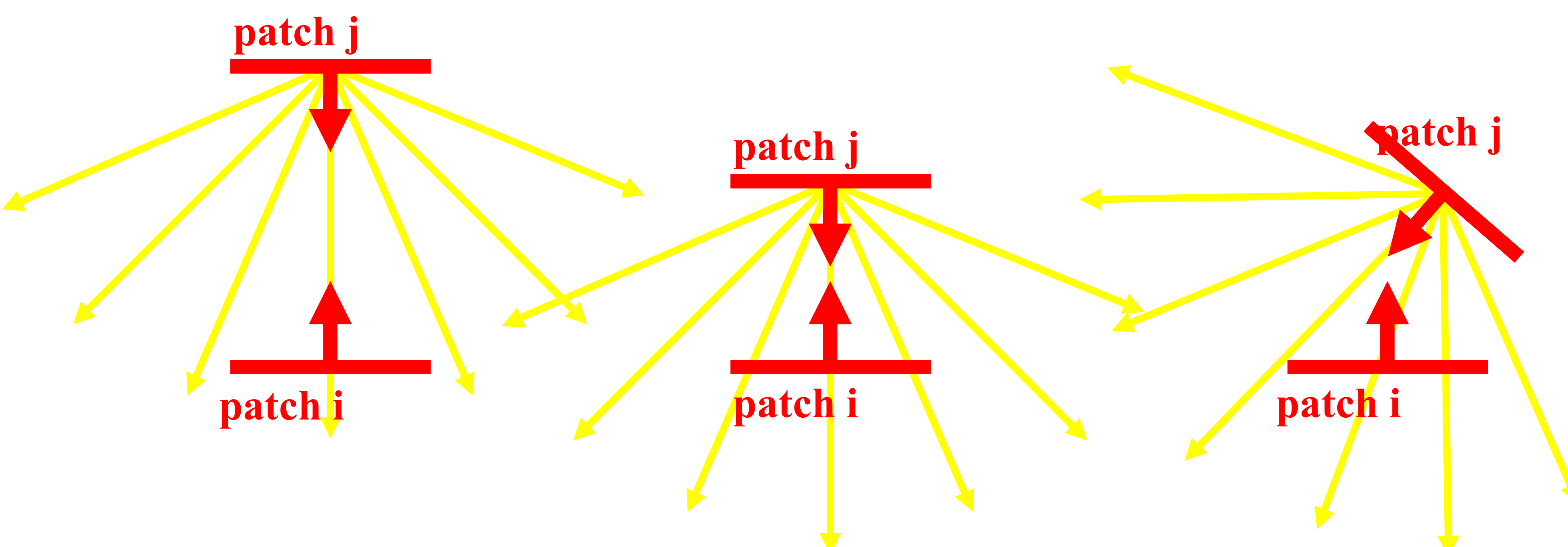
- B_i are constant per Element
- How to map to graphics hardware?
  - Average radiosity-values for each vertex
  - Extrapolate for vertices on the boundary





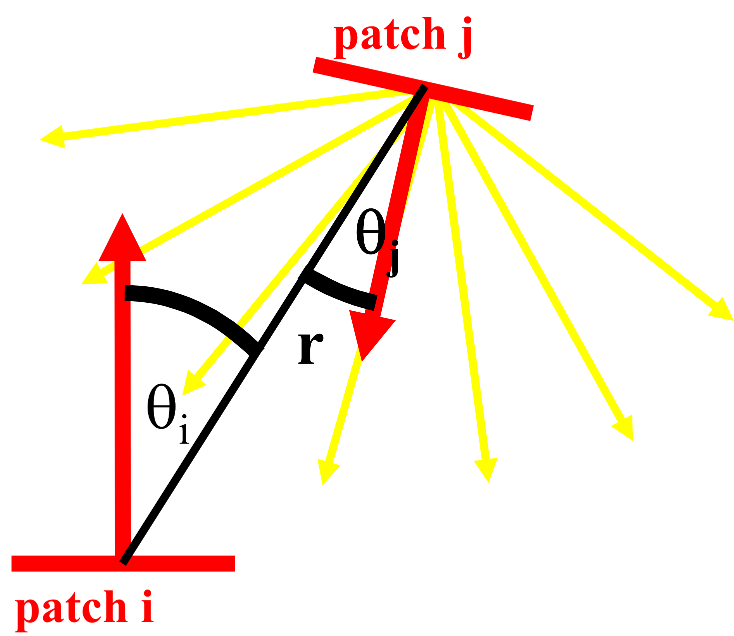
Form factors

- F_{ij} = Part of radiance from j that reaches i
- Influenced by:
  - Geometry (area, orientation, position)
  - Visibility (other elements of the scene)



Form factors

- F_{ij} = Part of radiance from j that reaches i

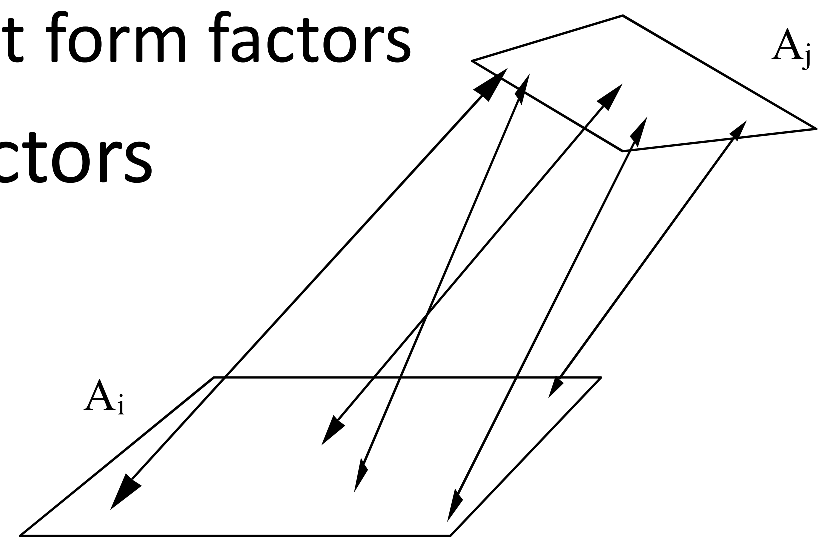


$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi r^2} V_{ij} \, dA_j \, dA_i$$

Form factors

Ray casting

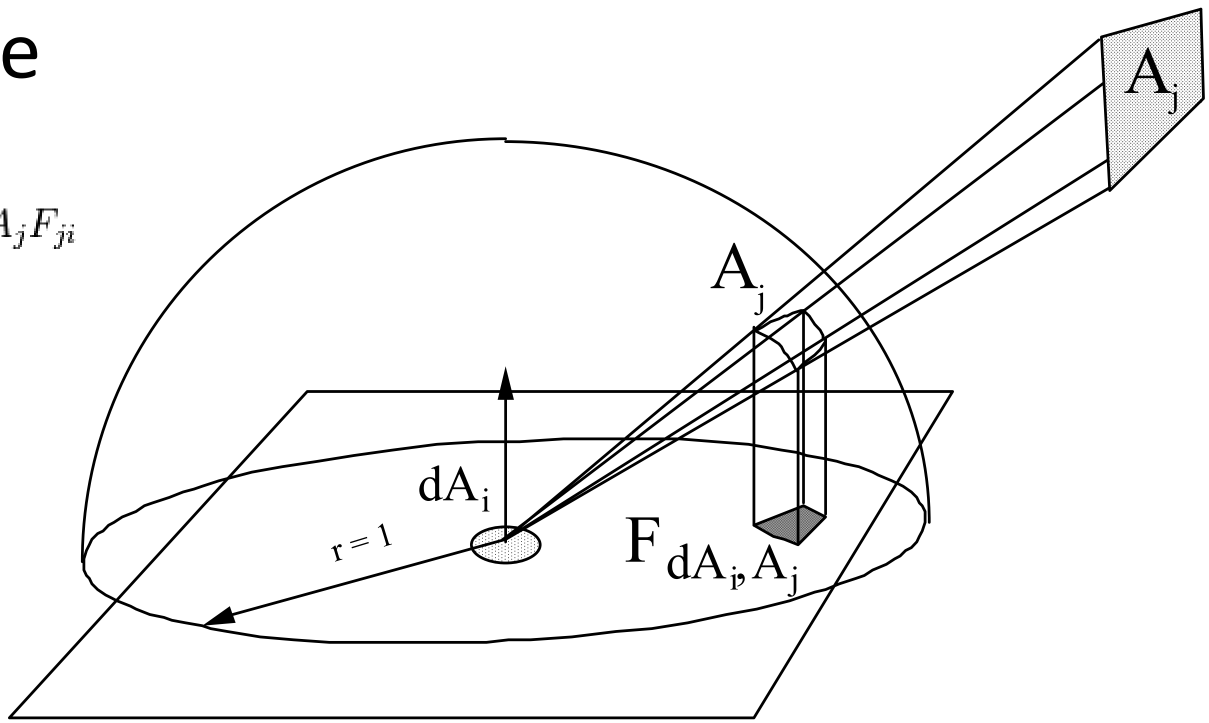
- Create *n* rays between 2 elements
  - *n* typically between 4 und 32
  - Determine visibility
  - Integrate point-point form factors
- Determines form factors between elements



Form factors

- Nusselt analog: the form factor is equivalent to the part of the unit circle, which the projection of the element occupies on the unit sphere

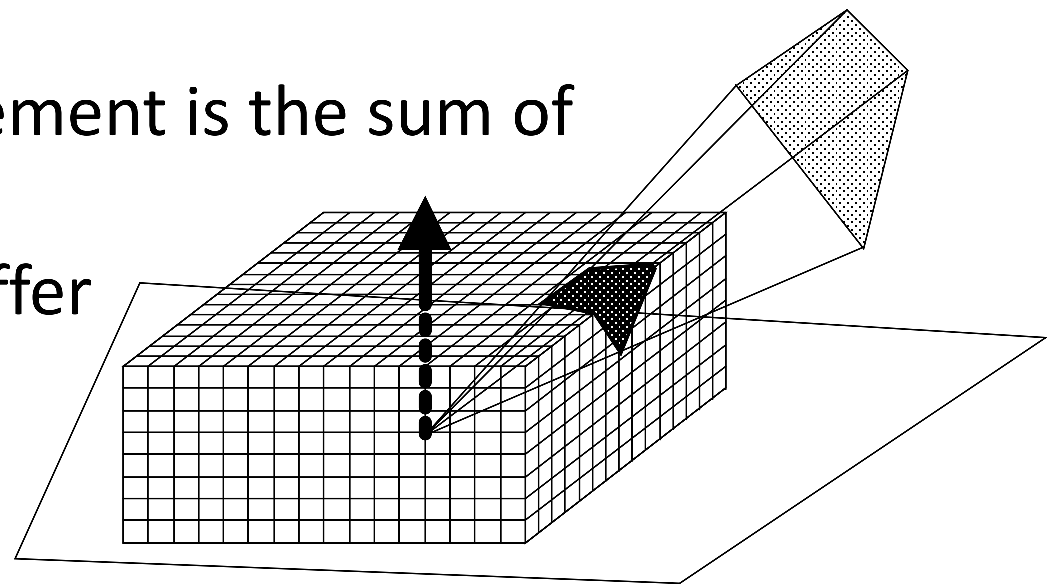
$$A_iF_{ij} = A_jF_{ji}$$



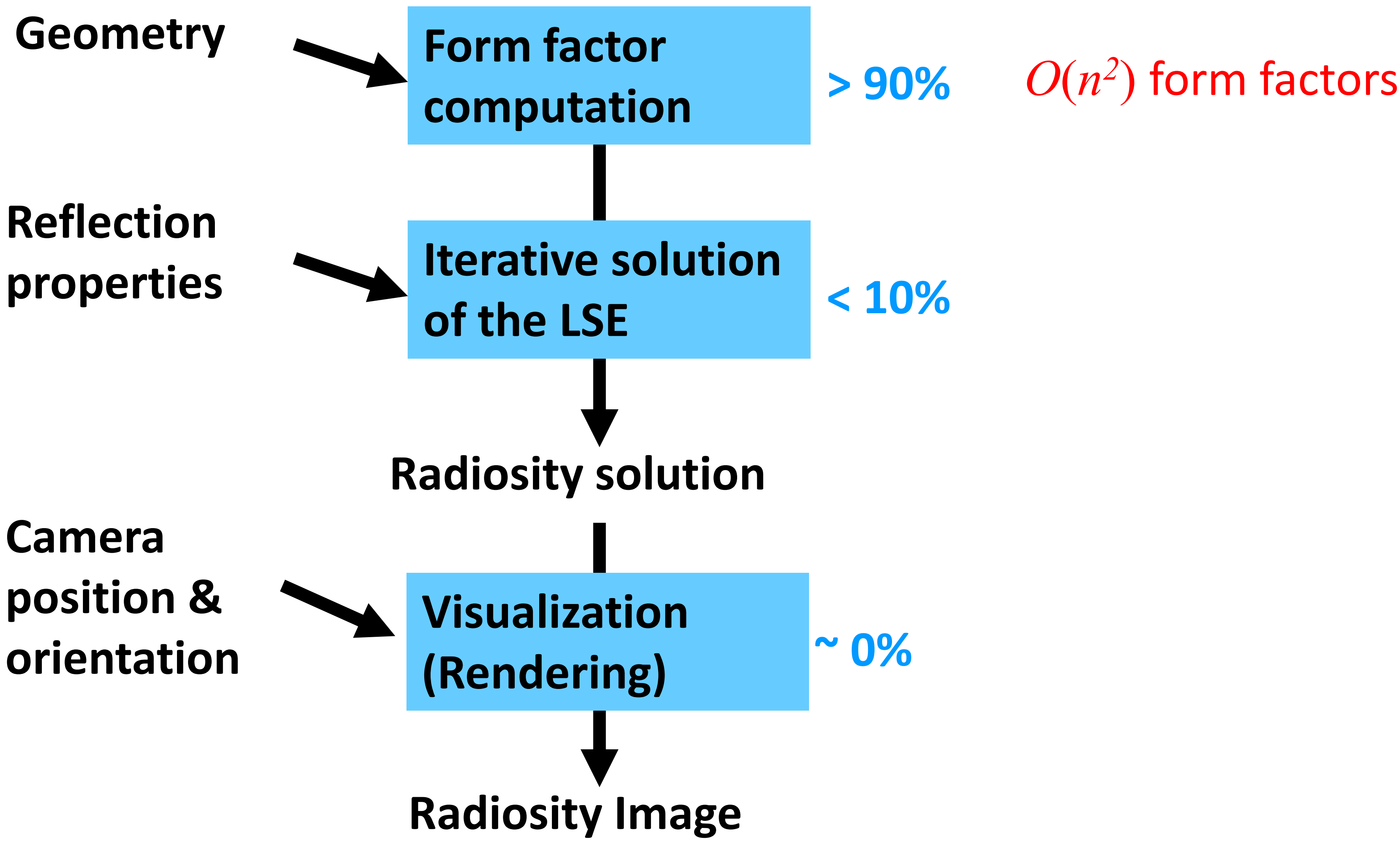
Form factors

Hemicube algorithm

- Place hemicube at element center
- Discretize the sides into pixels
- Project and rasterize other elements into cube
- Each hemicube pixel contains precomputed form factor
- Form factor for an element is the sum of contributions
- Visibility by depth buffer

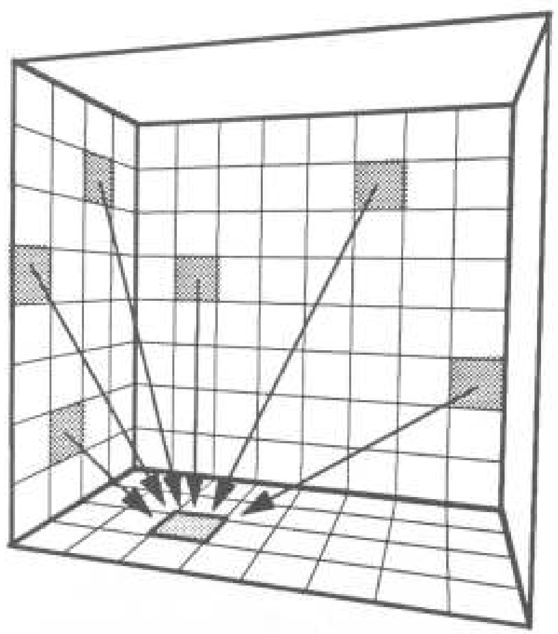


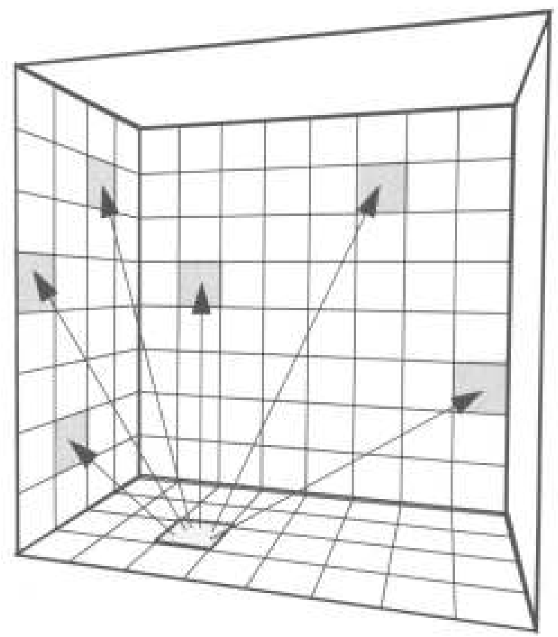
Solving the radiosity equation



Progressive refinement

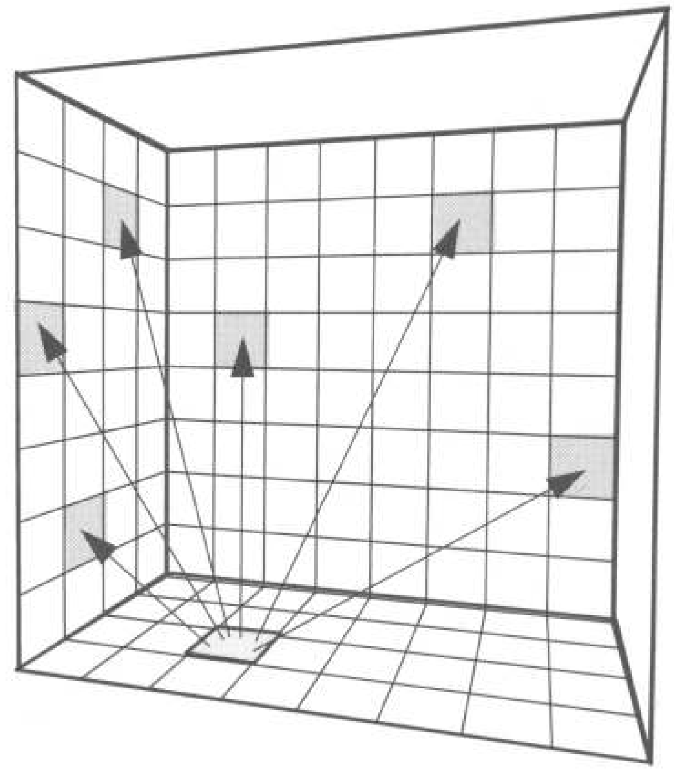
- Idea: instead of collecting radiosity from all sources ("gathering"), rather distribute radiosity from brightest emitters ("shooting")





Progressive refinement

$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ \\ \vdots \\ B_n \end{bmatrix} + \begin{bmatrix} \cdots & \rho_1 F_{1i} & \cdots \\ \cdots & \rho_2 F_{2i} & \\ & & \\ & & \\ & & \\ \cdots & \rho_n F_{ni} & \cdots \end{bmatrix} \begin{bmatrix} \\ \\ \vdots \\ B_i \\ \vdots \\ \\ \end{bmatrix}$$



Progressive refinement

- Each patch has remaining radiosity ΔB_i
- Start with $B_i=E_i$ and $\Delta B_i = E_i$
- Distribute ΔB_i to the scene
- Reciprocity:

$$B_i = E_i + r_i \sum_{j=1}^{n} B_j F_{ij}, \quad \text{for all } i$$

$$A_j F_{ji} = A_i F_{ij}$$

$$B_i = E_i + r_i \sum_{j=1}^{n} B_j F_{ji} \frac{A_j}{A_i}$$

Progressive refinement

- After sending from patch j, the radiosity of elements A_i is increased

$$B_i = B_i + r_i \Delta B_j F_{ji} \frac{A_j}{A_i},\ i = 1..n$$

- The nondisribuited radiosity is also increased

$$\Delta B_i = \Delta B_i + r_i \Delta B_j F_{ji} \frac{A_j}{A_i},\ i = 1..n$$

- The set undistributed radiosity of j to zero

$$\Delta B_i = 0$$

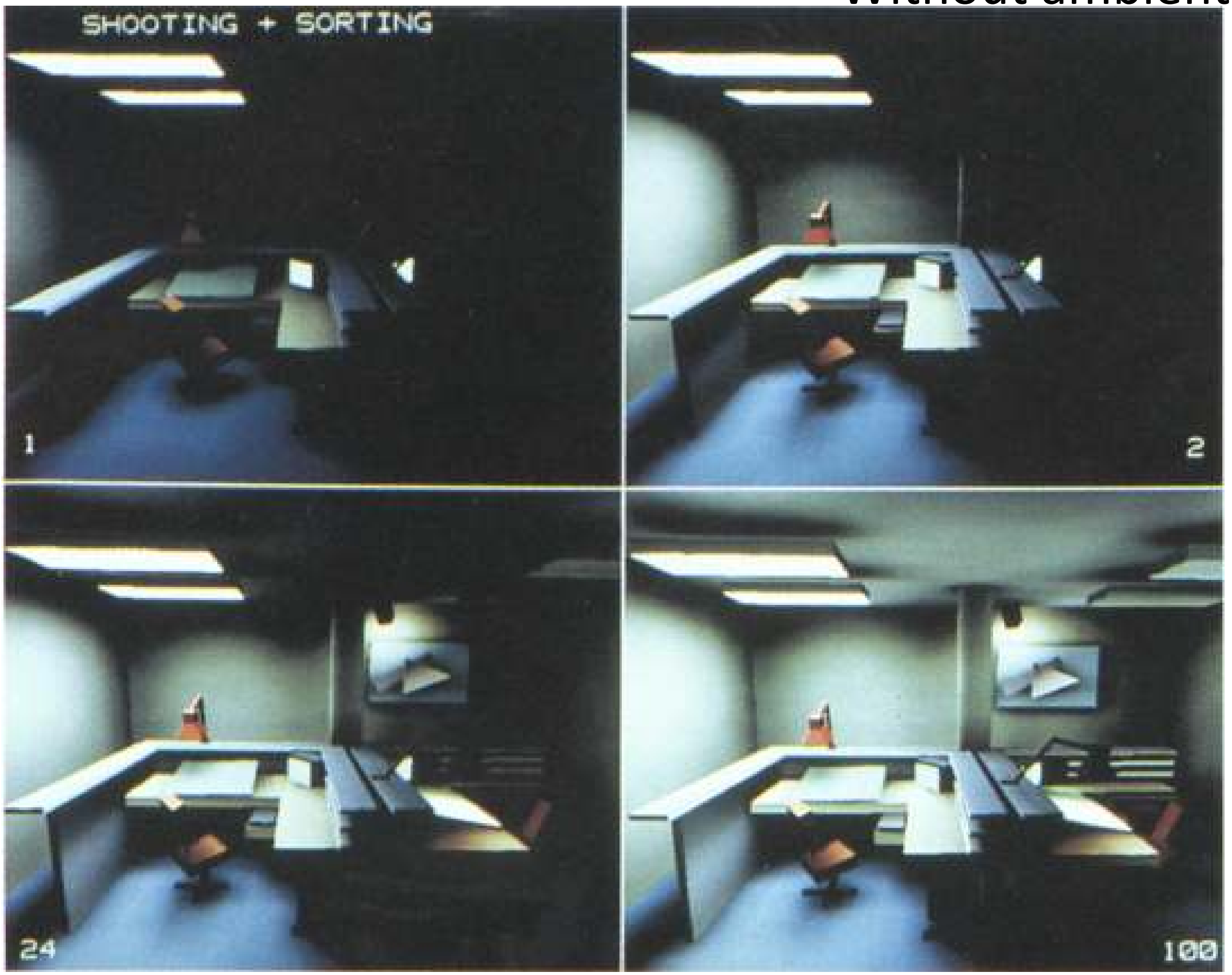
Progressive refinement

Advantages

- Each iteration only requires form factors F_{ij} for element i w.r.t. all other patches
- Good results after few iterations, resulting in significantly less overhead when compared to Gauss-Seidel iterations
- Only requires storing a single column of the form factor matrix

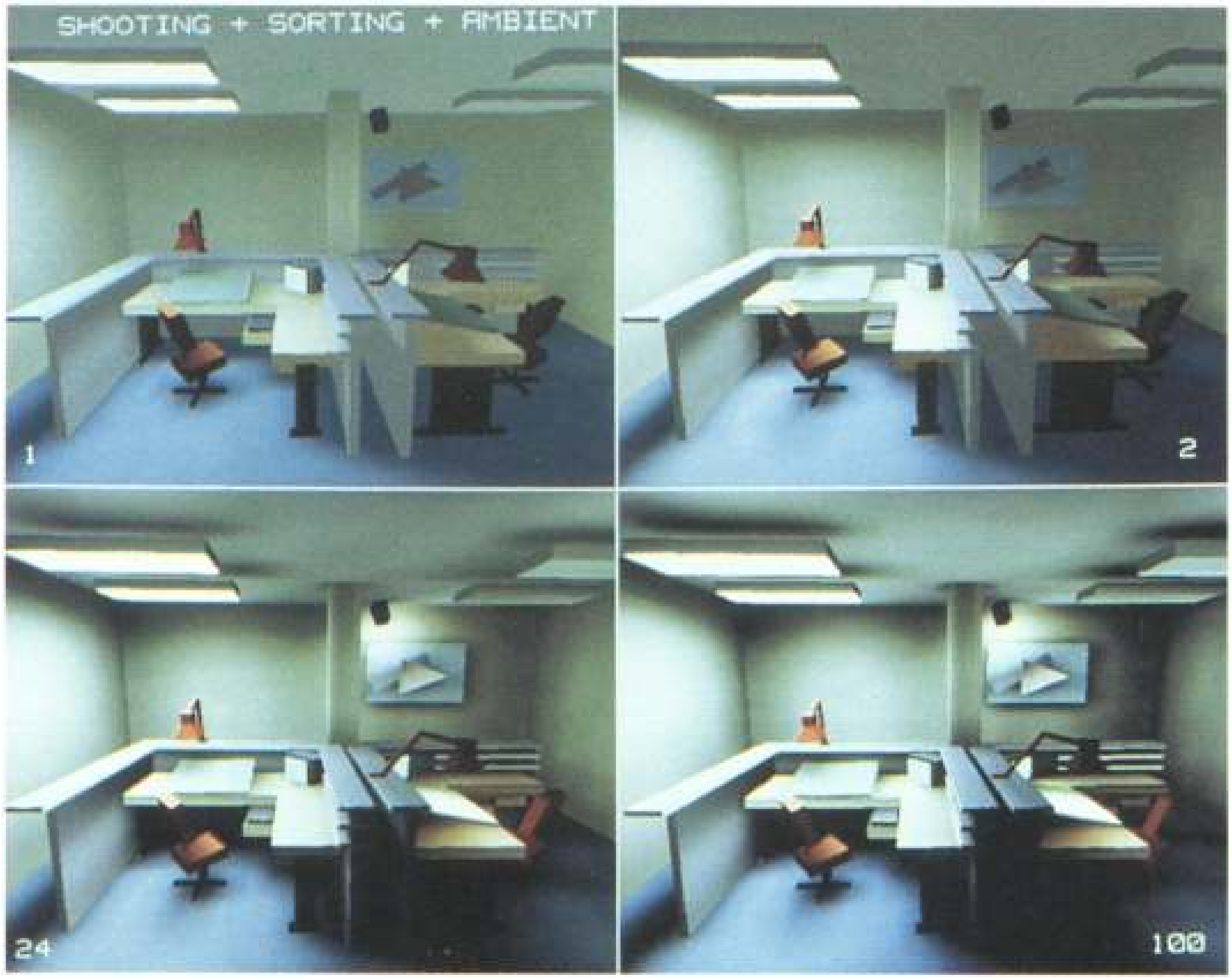
Progressive refinement

Without ambient term



Progressive refinement

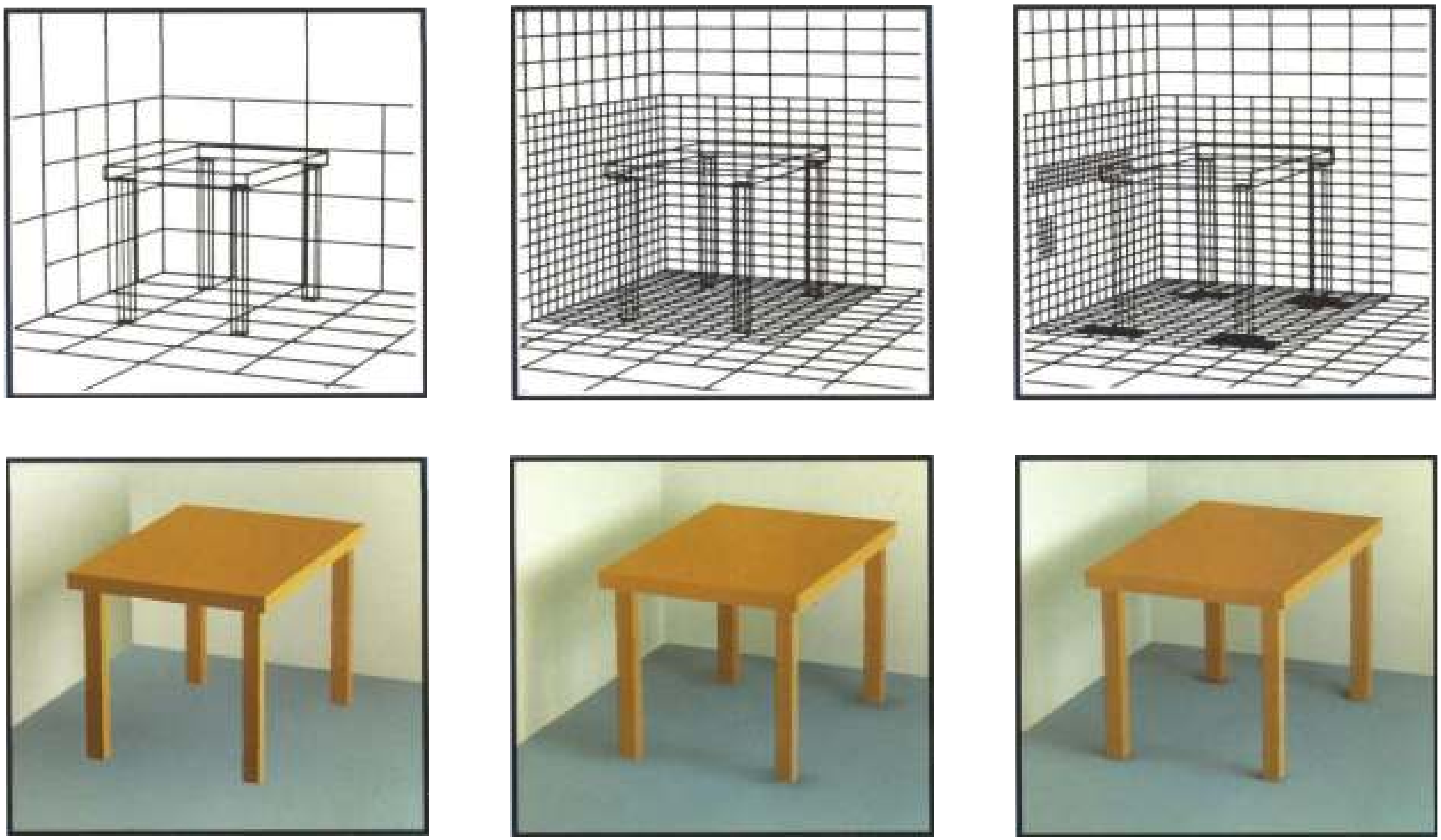
With ambient term



Discretization into patches

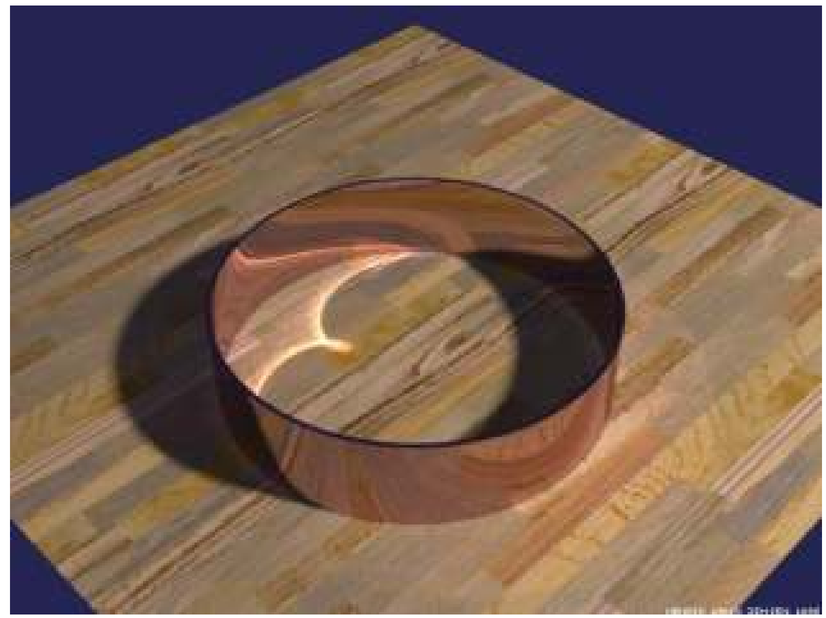
- Image quality depends on the size of patches
  - Smaller patches – smaller error
- Patches should be adaptively subdivided where large gradients in radiosity are evident
  - Start with regular grid
  - Subdivide based on quality criterion

Discretization into patches



Photon Mapping

Jensen 95



Examples

Lightscape **`http://www.lightscape.com`**

Examples

Mental Ray