

CS 428: Fall 2009

Introduction to Computer Graphics

Computer animation

Animation

A brief history



- 1800s – Zoetrope
- 1890s – Start of film animation (“cells”)
- 1915 – Rotoscoping
 - Drawing on cells by tracing over live action
- 1920s – Disney
 - Storyboarding (for story review)
 - Camera stand animation (parallax etc.)



Animation

A brief history

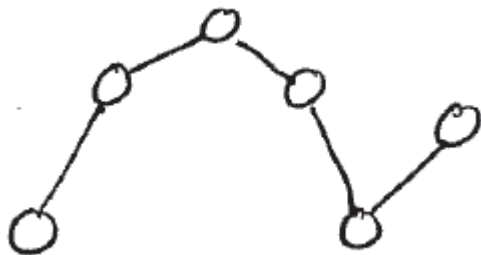
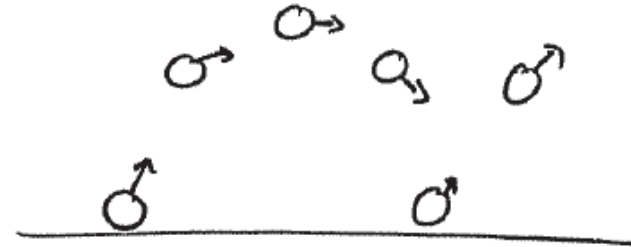
- 1960s – Early computer animation
- 1986 – Luxo Jr.
- 1987 – John Lasseter’s SIGGRAPH article
 - Applying traditional animation to CG animation (squash, stretch, ease in-out, anticipation, etc.)
- Before this
 - Tron (1982), Star wars (1977), etc.
- After this: artists needs became important!
 - Artists need a way of defining motion

Interpolation

- Interpolation of
 - Object/world geometry (positions)
 - Object/world parameters (angles, colors)
 - Object/world properties (lights, time of day)
- But what to interpolate between?
- Basic idea: **keyframe interpolation**
 - Sparse specification of *key* moments of an animation sequence

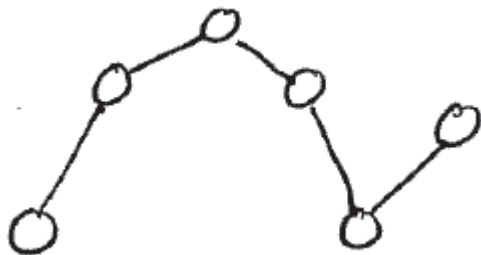
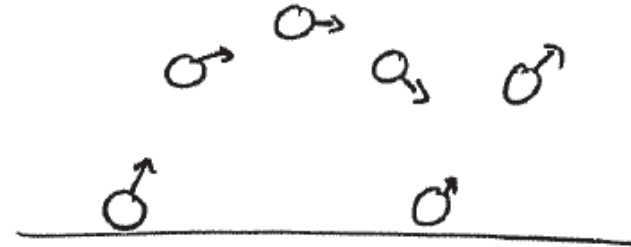
Keyframe interpolation

- Position / configuration
- Time of event
- Optional: velocity, acceleration, etc.
- Generate “in betweens” automatically
 - Interpolated motion paths are not unique



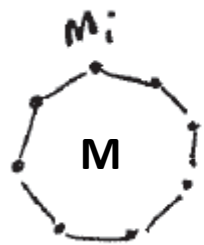
Keyframe interpolation

- Position / configuration
- Time of event
- Optional: velocity, acceleration, etc.
- Generate “in betweens” automatically
 - Linear and/or splines (keyframes at the knots)



Interpolation examples

- **Tweening** – interpolate from one mesh to another with some mesh connectivity



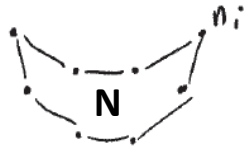
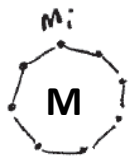
- Interpolate vertices

$$A = M \cdot (1-t) + N \cdot (t)$$

$$a_i = m_i \cdot (1-t) + n_i \cdot (t)$$

↑
not time





Interpolation examples

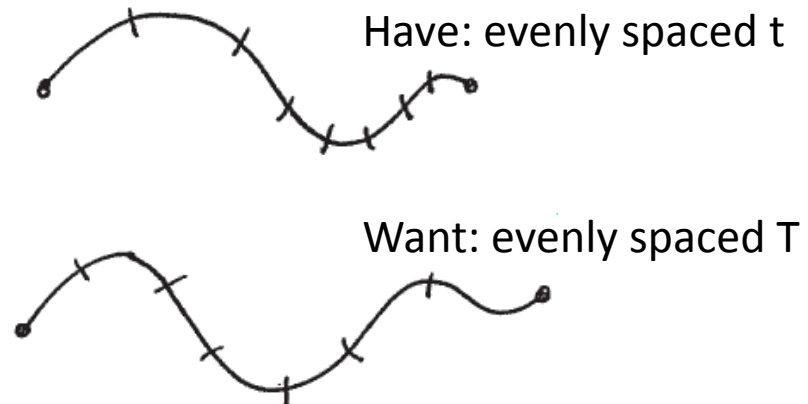
- Time warping – adjust time to influence anim

$$\begin{array}{l}
 M \text{ at } T_1 \\
 N \text{ at } T_2
 \end{array}
 \quad
 f(t) = T_1 + (T_2 - T_1)t
 \quad
 \begin{array}{l}
 f(0) = T_1 \\
 f(1) = T_2
 \end{array}
 \quad
 \begin{array}{l}
 f^{-1}(T_1) = 0 \\
 f^{-1}(T_2) = 1
 \end{array}$$

$$A(T) = M \cdot (1 - f^{-1}(T)) + N \cdot f^{-1}(T)$$

- Perhaps use a spline to represent f

- Gives animator more control
- Move knots for an arc-length parameterization



Interpolation examples

- Simple linear interpolation in (pseudo) code

```
double h(double t)
```

```
if (t < t1)  
    return h1
```

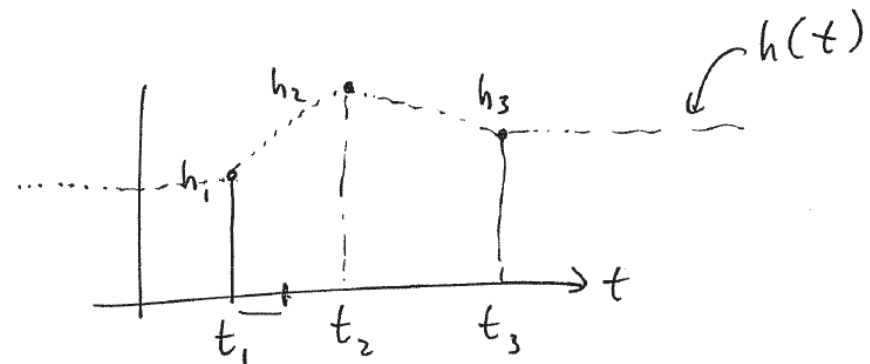
```
if (t < t2)
```

```
    return h1 +  $\frac{t-t_1}{t_2-t_1} (h_2-h_1)$ 
```

```
if (t < t3)
```

```
    return h2 +  $\frac{t-t_2}{t_3-t_2} (h_3-h_2)$ 
```

```
return h3
```



Interpolating parameters



$s = 0$



$s = 1$

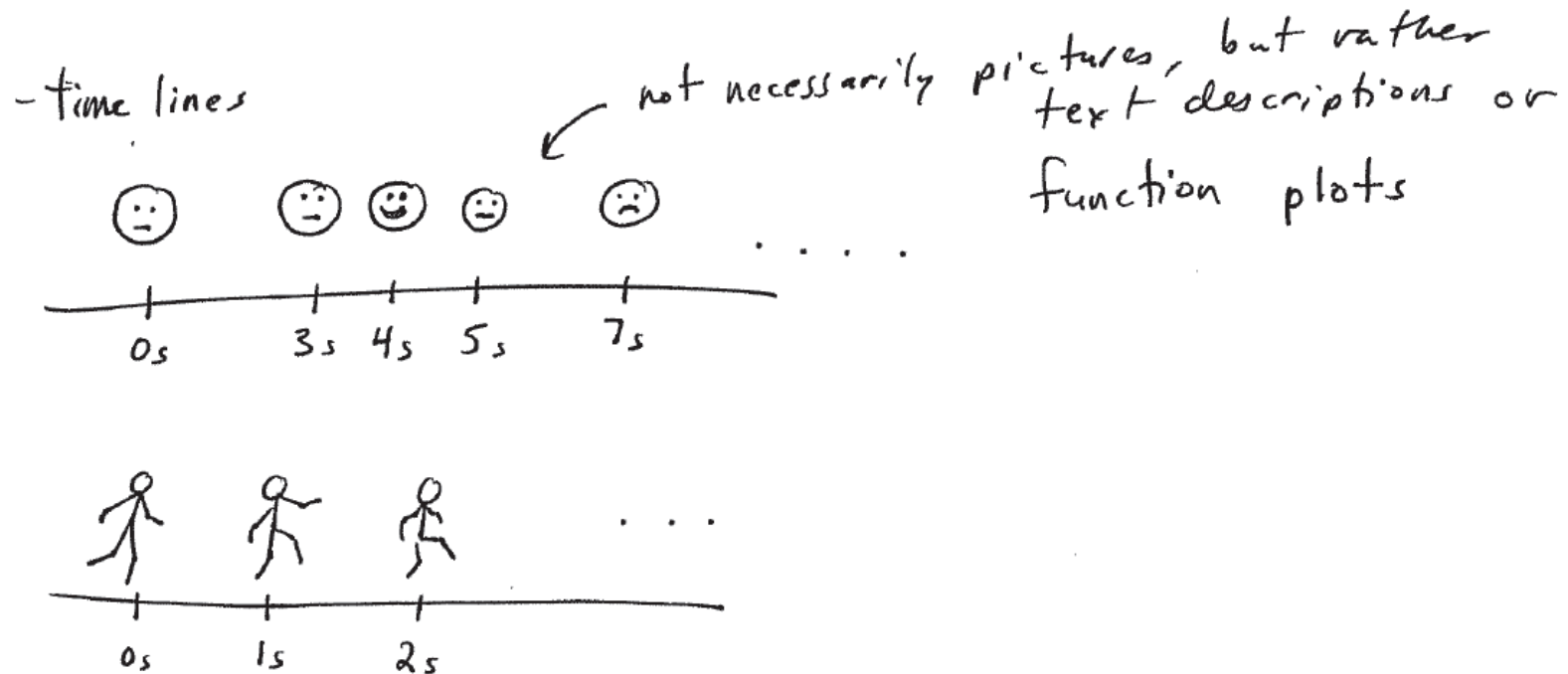


$s = -1$

- Interpolate s as before
- Interpolating rotation angles can be tricky
 - Euler angles $R_x(\alpha_x)R_y(\alpha_y)R_z(\alpha_z)$
 - Counterintuitive + erratic for distant keyframes
 - Use quaternions instead

$$[s, a_x, a_y, a_z] \quad \text{where} \quad s^2 + a_x^2 + a_y^2 + a_z^2 = 1$$

User interfaces for keyframes

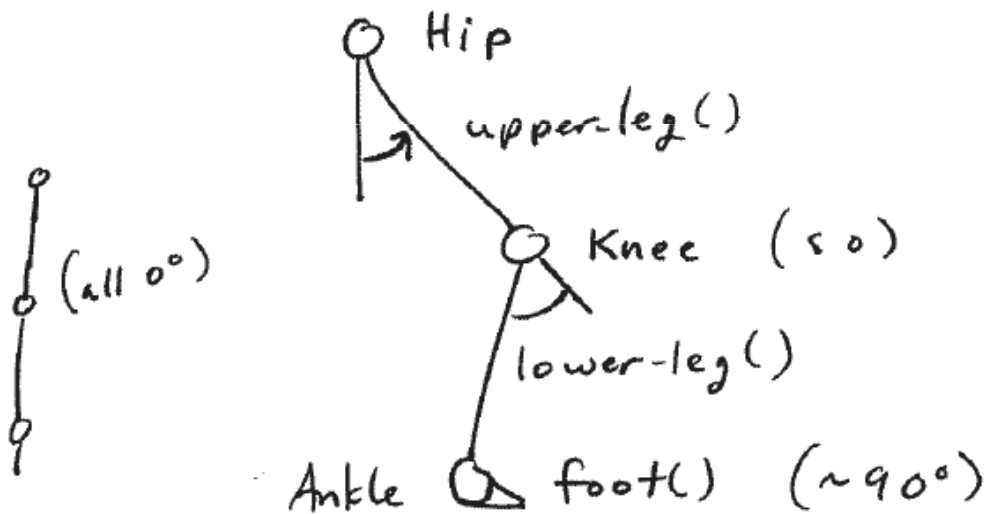


Typically 1 in 5 frames (in 30 frames/sec) are keyframes

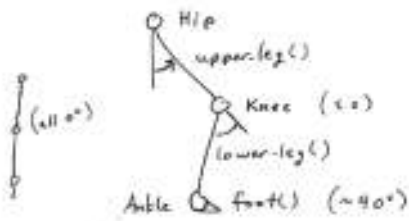
→ more than cell animation, since no human there with good judgement

Timelines /w function plots

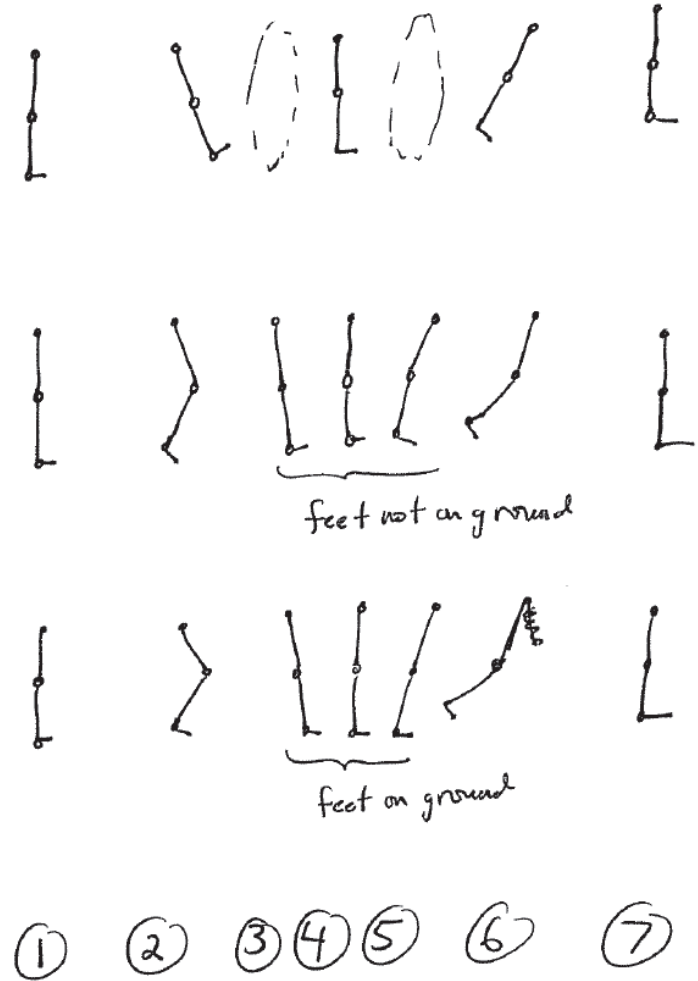
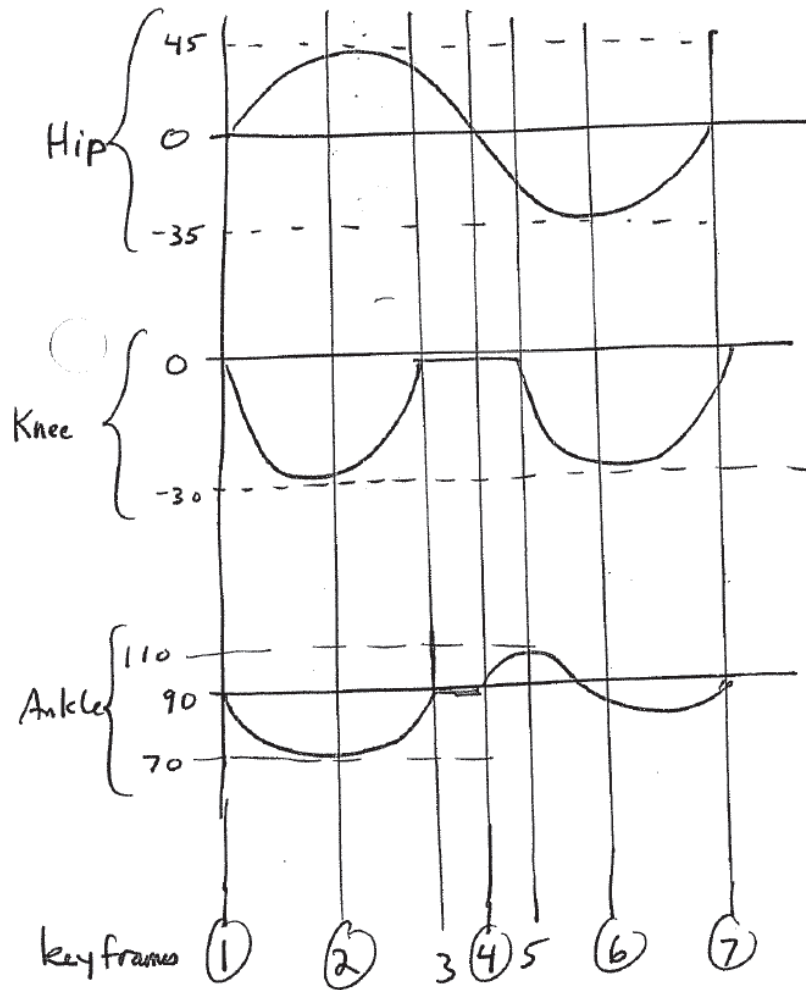
- Leg example



```
rotate(Hip, 0, 0, 1)
upper-leg()
rotate(Knee, 0, 0, 1)
lower-leg
rotate(Ankle, 0, 0, 1)
foot()
```

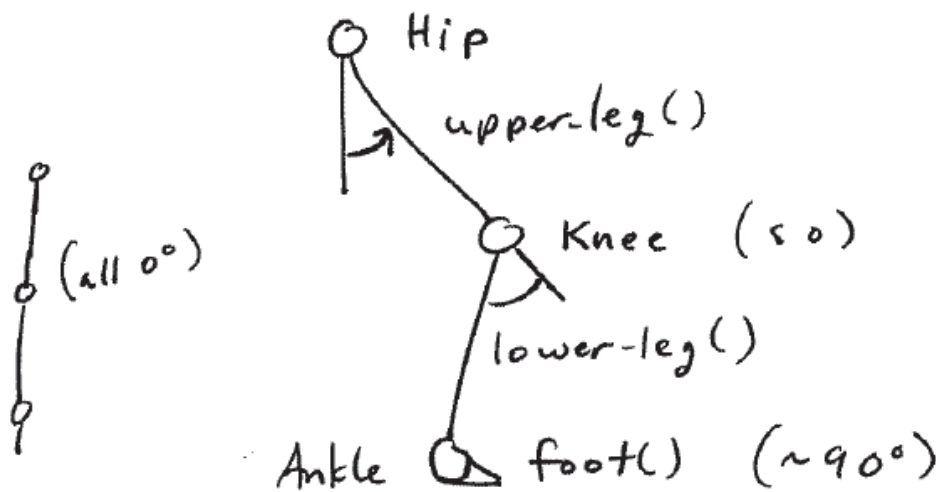


Timelines /w function plots



Timelines /w function plots

- A lot of work!
- Even worse: ankle depends on hip + knee!
- Kinematics: animation/w motion parameters (pos, vel, accel). No reference to forces



```
rotate(Hip, 0, 0, 1)
upper-leg()
rotate(Knee, 0, 0, 1)
lower-leg
rotate(Ankle, 0, 0, 1)
foot()
```

Physically based animation

- Each moving object is a point in a force field
 - Position and velocity
 - **Acceleration:** computed from the environment and integrated over time to determine pos + vel

$$\frac{d \begin{pmatrix} v \\ x \end{pmatrix}}{dt} = \begin{pmatrix} a \\ v \end{pmatrix}$$

Euler integ (given $a(t)$)

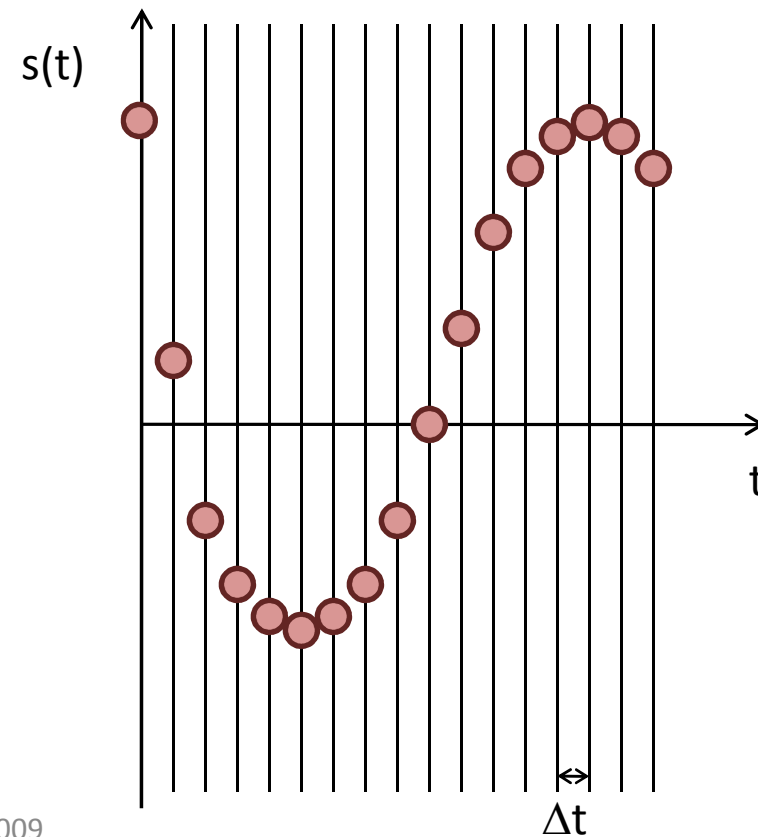
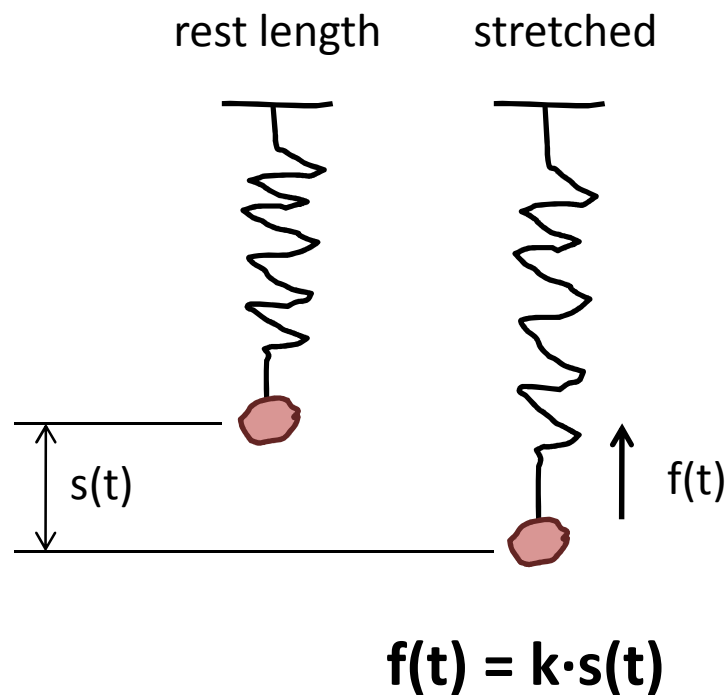
$$v(t + \Delta t) = v(t) + a(t) \Delta t$$

$$x(t + \Delta t) = x(t) + v(t) \Delta t$$

- $\mathbf{f} = \mathbf{m} \cdot \mathbf{a}$ (or $\mathbf{a} = \mathbf{f}/\mathbf{m}$) \rightarrow Newton's 2nd law
- Careful about choice of Δt !

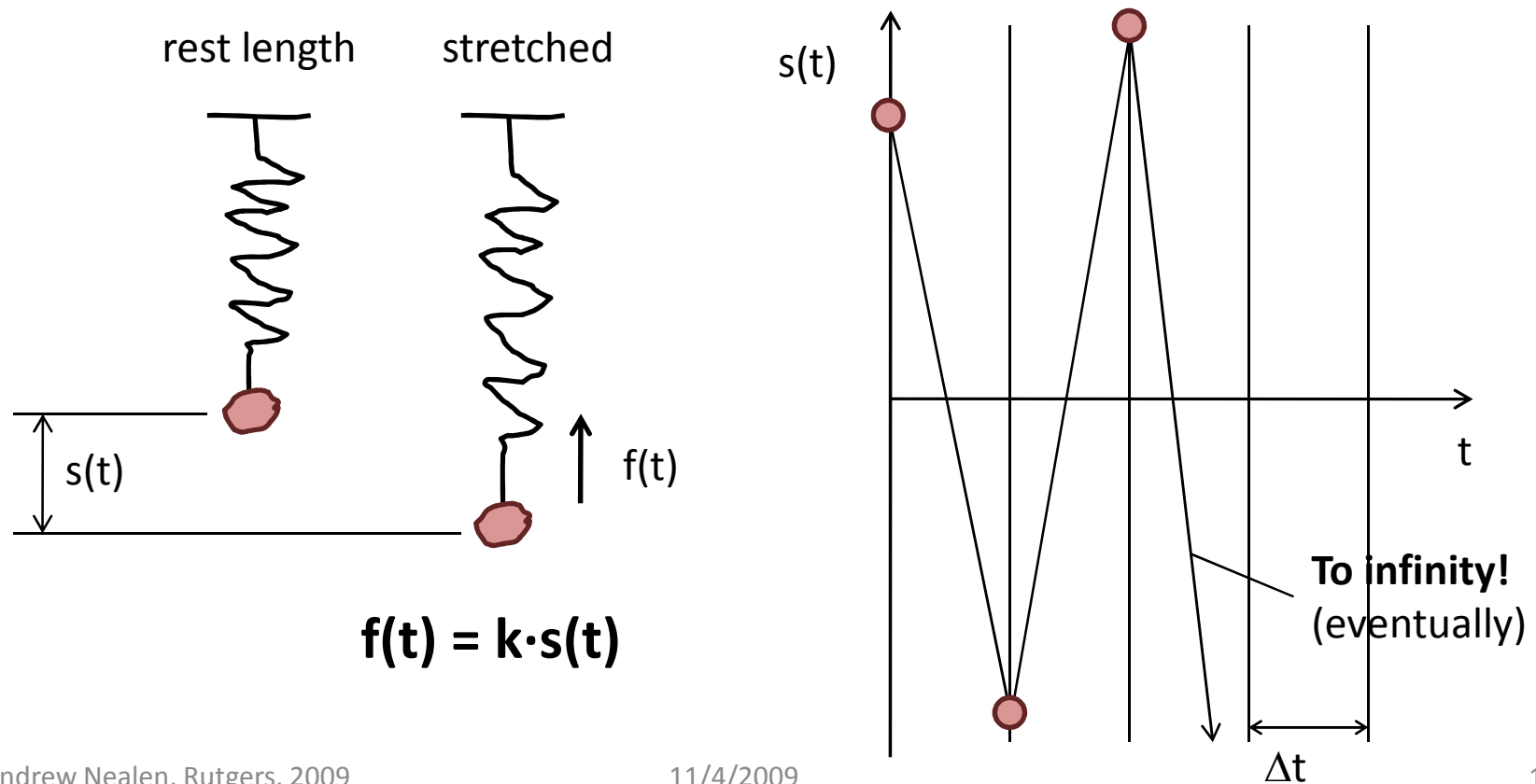
Physically based animation

- Time step in Euler integration
 - Depending on stiffness of ODE, **smaller time step**



Physically based animation

- Time step in Euler integration
 - Depending on stiffness of ODE, **smaller time step**




Accelerations

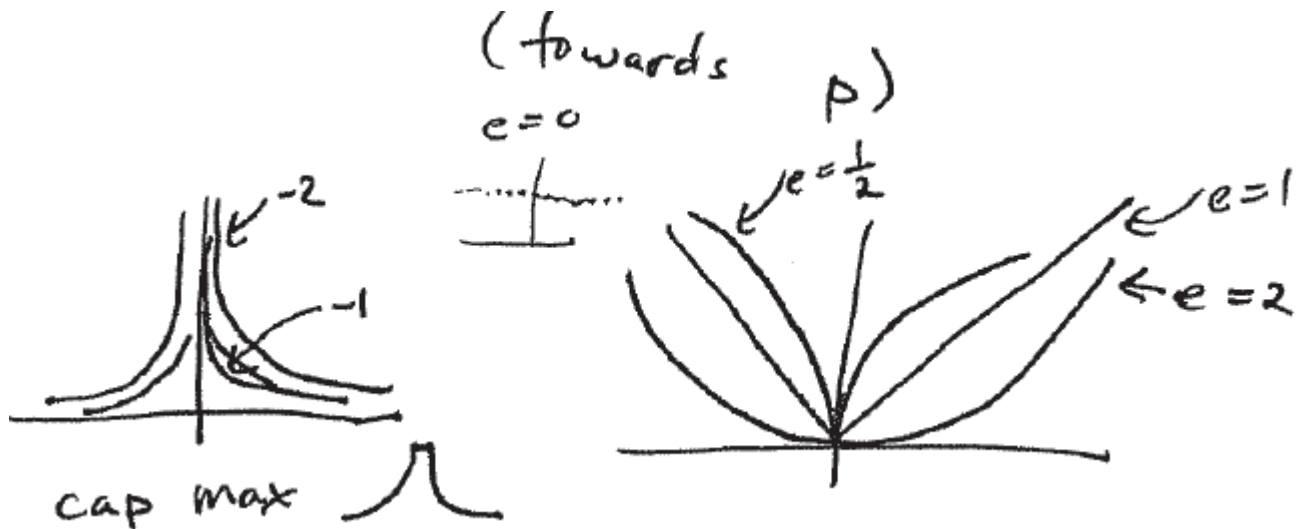
- Distance based Attraction + repulsion forces

$$a = k \frac{1}{d} \frac{1}{\|d\|^e}$$

scaling constant \nearrow k \nwarrow power e

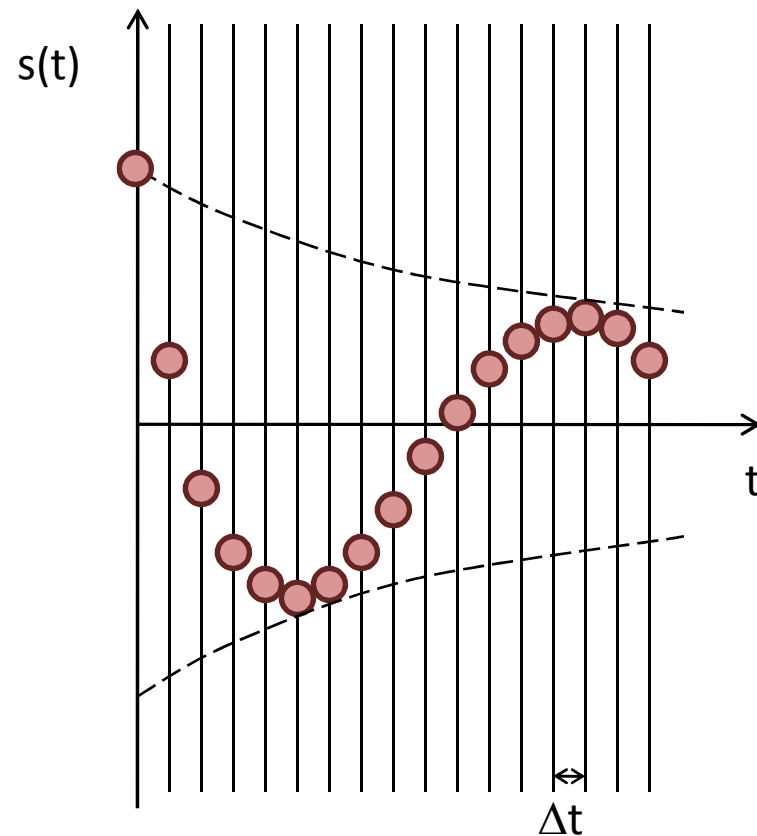


$k > 0 \Rightarrow$ attraction ; $k < 0 \Rightarrow$ repulsion



Accelerations

- Viscous drag $a = -k v$
- Numerical stability
- Linearly depends on velocity
 - Air drag
 - Drag inside a liquid
 - k depends on medium in which object is immersed



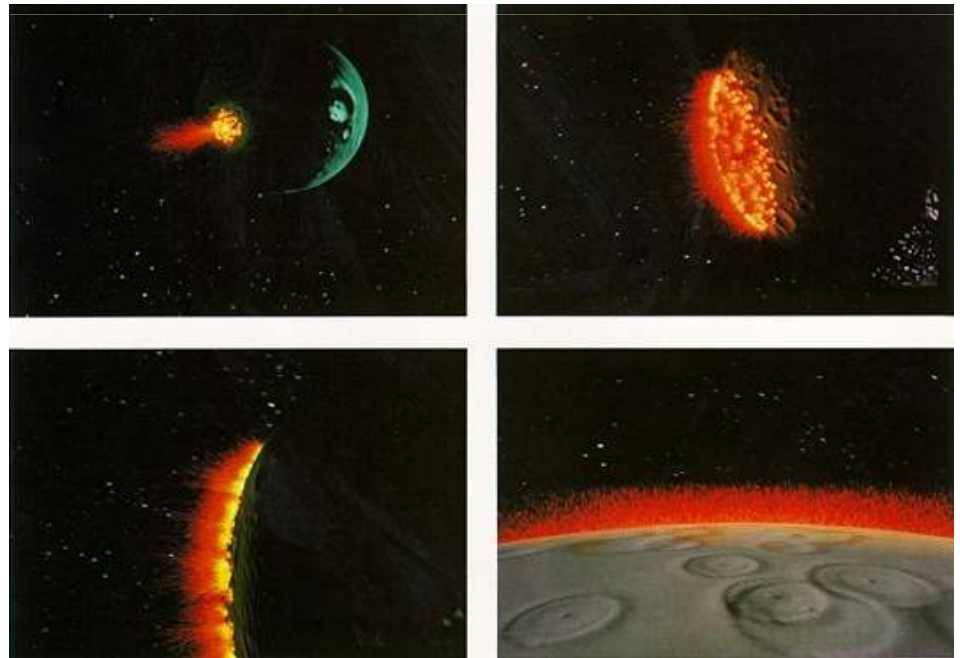
Simulation loop

- Sum up all accelerations per point at time step t
 - Springs
 - Gravity
 - Attraction + repulsion
- Perform one step of Euler integration
 - Obtain updated velocity and position at time step $t+1$
- Repeat

$$a_{total} = \sum a$$

Particle systems

- For modeling moving, amorphous phenomena
 - Fire, gas, water, explosions
- Collection of particles, where each has
 - Initial position and velocity
 - Initial size, shape, transparency
 - Shape
 - Lifetime
 - Etc.



$$f_{\text{avoid}} = \frac{(p - \text{obj})}{\|p - \text{obj}\|} \cdot k \|p - \text{obj}\|^p$$

Behaviors

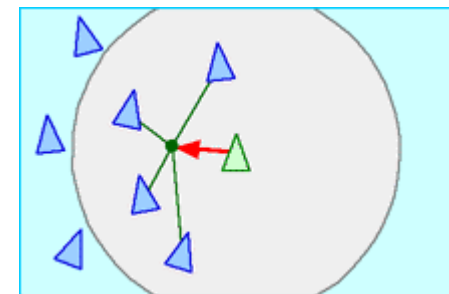
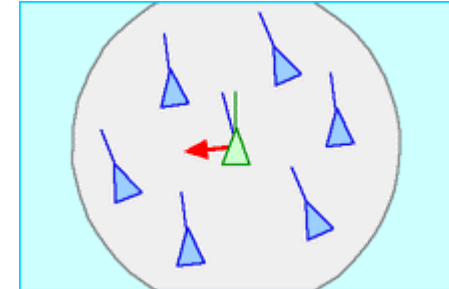
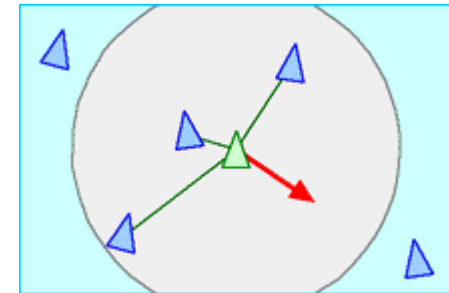
For higher level control

■ **Flocking:** three layered behaviors

- Separation / collision avoidance
 - Steer to avoid crowding flockmates

- Alignment / velocity matching
 - steer towards the average heading of local flockmates

- Cohesion / flock centering
 - steer to move toward the average position of local flockmates



Simulation

- Dynamics

$$f = ma = m \frac{d^2 x}{dt^2}$$

$a(t)$
- accel is a function
of time

$$v(t + \Delta t) = v(t) + \frac{f}{m} \Delta t$$

$$x(t + \Delta t) = x(t) + v(t) \Delta t$$

a, v, \bullet are vectors } \mathbb{R}^3
 x is a position }

- Forces: gravity, viscous drag, attraction, etc.
- Collision detection + response?
- Animator control?

Alternatives

- Closed form solutions

bouncing ball:

$$y(x) = A \left| \sin(\omega x + \phi_0) \right| e^{-kx}$$

amplitude angular freq phase damping



- Not always available

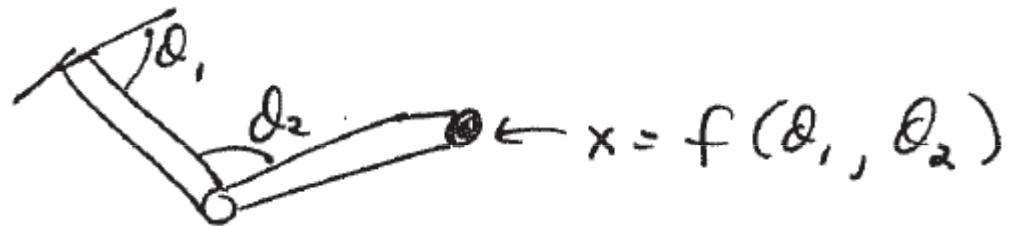
- Leg motion or walking is too complicated

Alternatives

- Don't use keyframes, but instead **constraints**

- “Keep foot flat on floor from frame 3-5”

- “Elbow/hand is at position x ”



- Given x , solve for θ_1 / θ_2 : **inverse kinematics**

- Use of nonlinear equations solvers

- Problems: non-uniqueness

- Gets worse with more degrees of freedom (DOFs)

- Use objective functions $E(\theta_1, \theta_2)$ and nearby solutions

