CS 428: Fall 2010

Introduction to Computer Graphics

Texture mapping and filtering

Topic overview

- Image formation and OpenGL
- Transformations and viewing
- **Polygons and polygon meshes**
  - **3D model/mesh representations**
  - **Piecewise linear shape approximations**
  - **Illumination and polygon shading**
- Modeling and animation
- Rendering

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  - **OpenGL rasterization: hidden surface removal, interpolation, texturing (all image-space!)**
- Modeling and animation
- Rendering

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  - Programmable pipelines
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- **Rendering**
  - **OpenGL rasterization: hidden surface removal, interpolation, texturing (some sampling theory)**
  - **Raytracing and radiosity**

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  - Programmable pipelines
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- **Rendering**
  - **Object space hidden surface removal, bump mapping and other texture tricks**
  - **Raytracing and radiosity**

Textures

- Reality provide a wide spectrum of geometric shapes and physical materials
  - Structures in wood, marble, wallpaper, clouds
  - Far-away background scenery, buildings and trees
- Modeling the exact geometric form of these objects is too complex and not necessary
  - No/small parallax effects for distant scenery + only cover a few pixels
  - Can be performed in a fragment shader (more later...)

Textures

- Make simple objects appear significantly more complex
- Model wall, mirror or window as planar surface
- Apply an image to the window to model the distant scenery

Texture mapping

- 2D-textures are functions that map points (u,v) in texture space to (r,g,b) colors

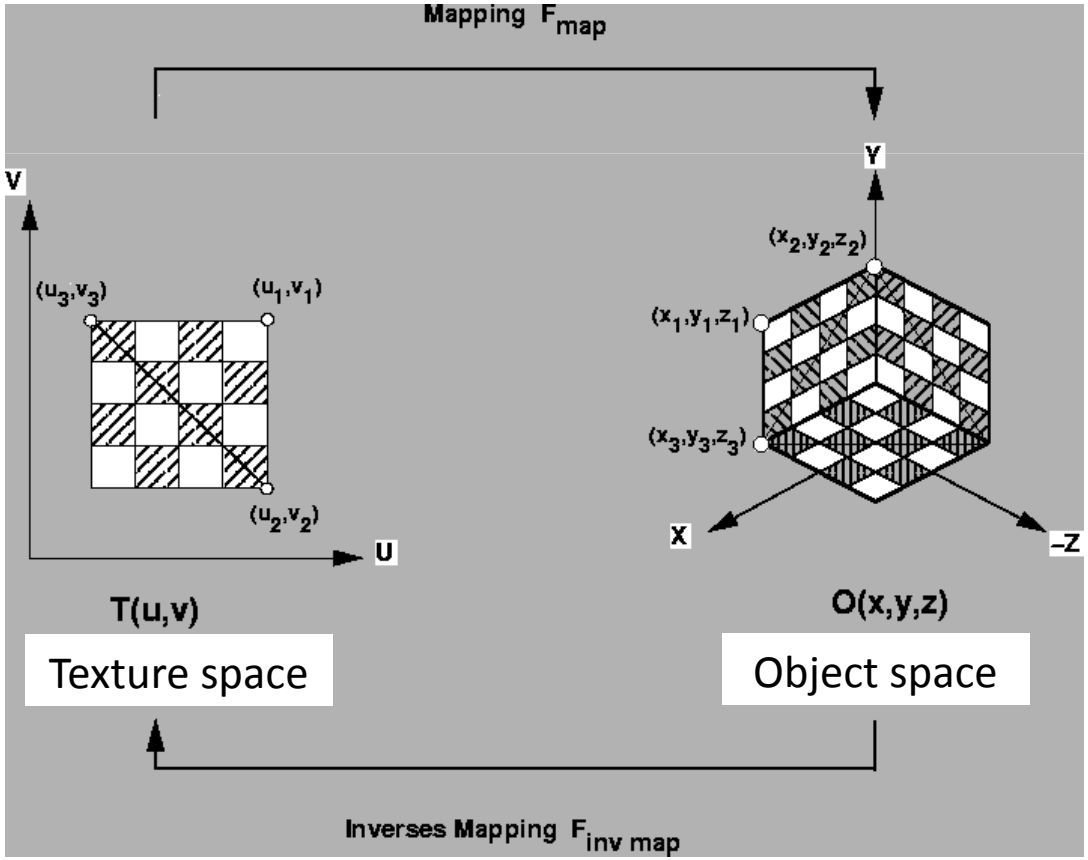
$$(r,g,b) = C_{tex}(u,v)$$

- The mapping describes how to decorate the surface
- For rendering, we need the inverse mapping from known (x,y,z) coordinates to (u,v) points

$$(u,v) = F_{inv\ map}(x,y,z)$$

Texture mapping

- Texture mapping uses barycentric coordinates
  - (u,v) provided at vertices **`glTexCoordXX(...)`**



Texture mapping

- Mathematically, this is described by concatenation of two mappings

$$(r,g,b) = C_{tex}(\overbrace{F_{inv\ map}(x,y,z)}^{2D})$$

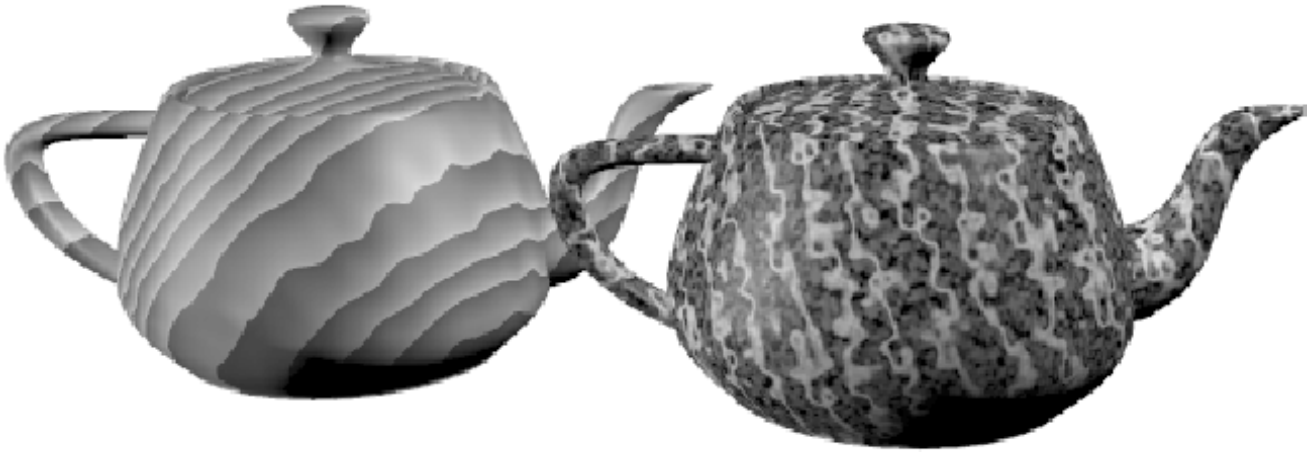
- 3D-textures are functions that map points of (u,v,w) space to (r,g,b) colors

$$(r,g,b) = C_{tex}(u,v,w)$$

Texture mapping

- 3D textures are known as **solid textures**
- Examples: wood and marmorite
- Interpretation: the shape is carved out of the (u,v,w) texture space

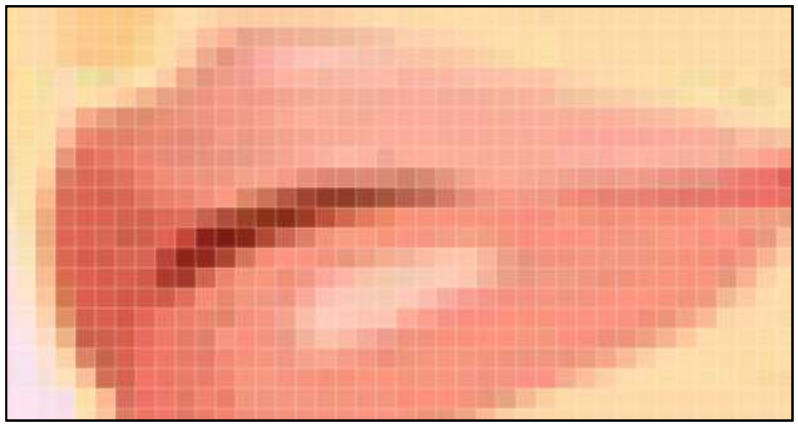
$$(r, g, b) = C_{tex}(u, v, w)$$



Texture representation

Discrete textures

- Grid of color values (texels)
- Three/four scalar quantities per texel (rgba)
- n x m texture is simply a pixel image with 3(4) tuples stored at integer coordinates

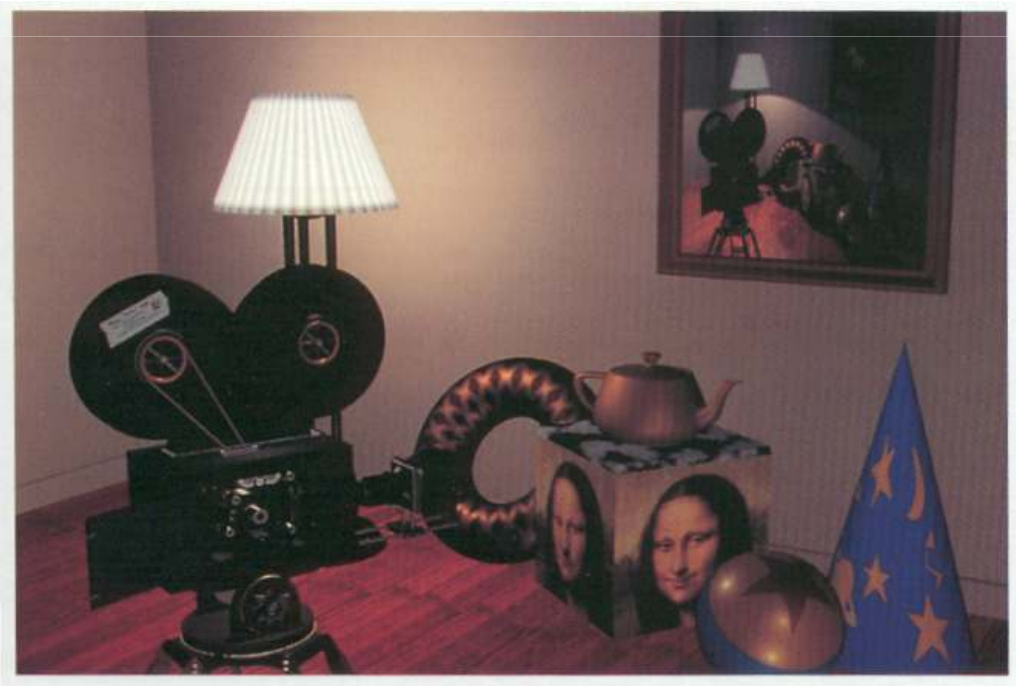


$$\{C[i,j] \mid 0 \le i < n,\ 0 \le j < m\}$$

Discrete textures

Advantages

- Easy to acquire with cameras, scanners etc.
- Generating complex textures using cameras and procedural methods simplifies photorealism



Discrete textures

Problems

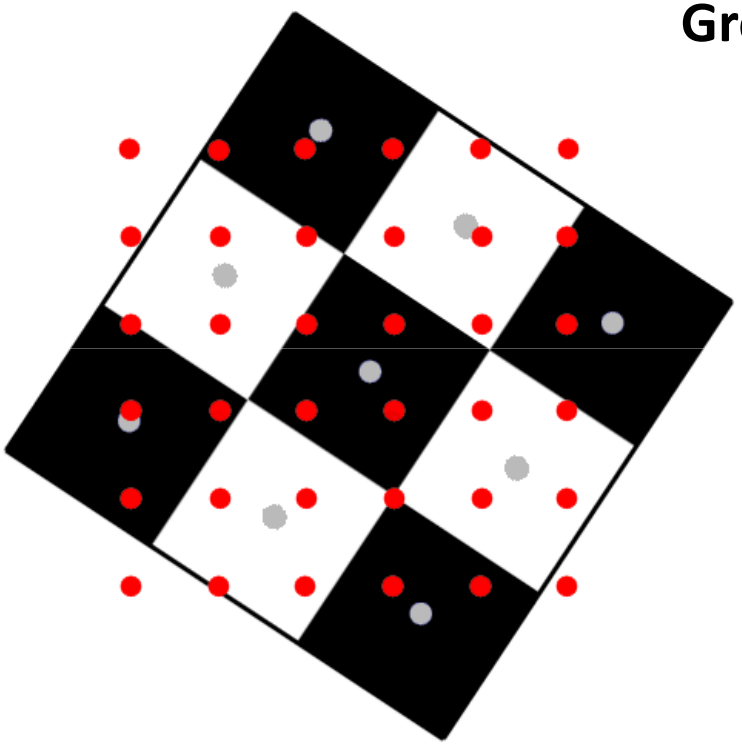
- Large texture memory consumption
- Magnifying textures leads to blurriness
- Tiling a surface with textures is complicated
- The texture lighting is not necessarily coherent with the lighting in the scene
- Texture mapping on surfaces is in general not free of distortion (parameterization, CS 523)
- Texture values $C_{tex}(u,v)$ at positions (u,v) need to be reconstructed, interpolated and filtered

Discrete textures

Reconstruction

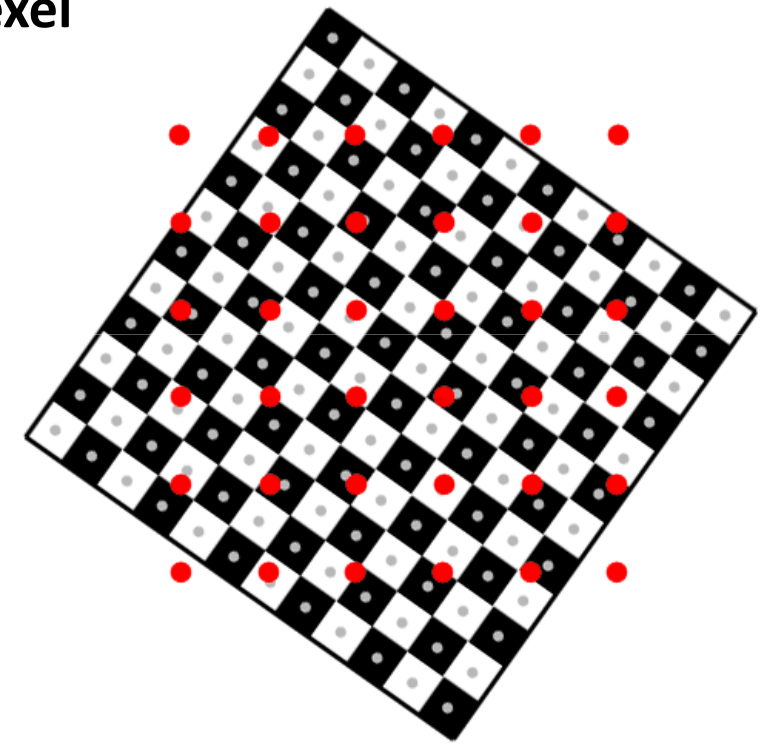
Red: screen pixel

Grey: texel



Magnification

(Oversampling)



Minification

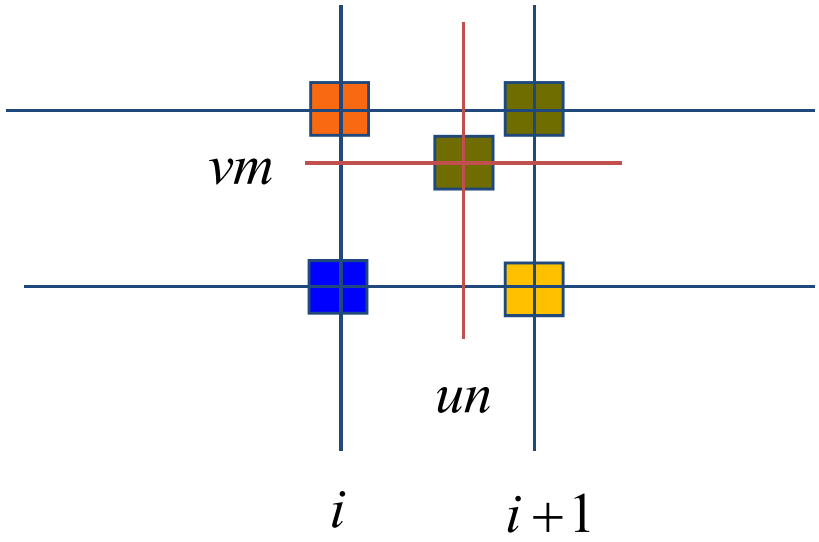
(Undersampling)

Discrete textures

Reconstruction

- 1. nearest neighbor filtering

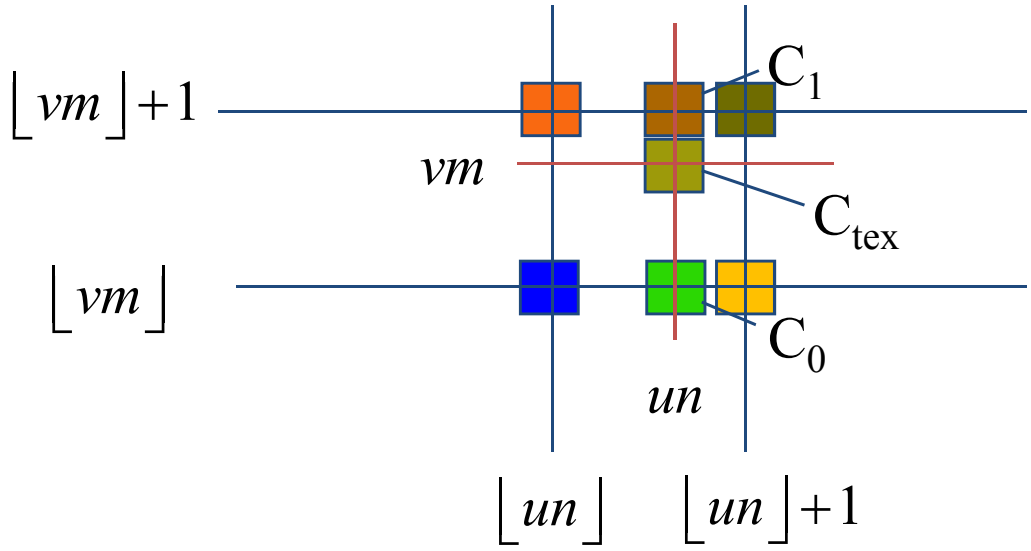
$$C_{tex}(u,v) = \begin{cases} C\left[\lfloor un \rfloor, \lfloor vm \rfloor\right] & : \text{u<1,v<1} \\ C\left[\lfloor n-1 \rfloor, \lfloor vm \rfloor\right] & : \text{u=1,v<1} \\ C\left[\lfloor un \rfloor, \lfloor m-1 \rfloor\right] & : \text{u<1,v=1} \\ C\left[\lfloor n-1 \rfloor, \lfloor m-1 \rfloor\right] & : \text{u=1,v=1} \end{cases}$$



Discrete textures

Reconstruction

- 2. bilinear interpolation



$$\hat{u} = un - \lfloor un \rfloor$$

$$\hat{v} = vm - \lfloor vm \rfloor$$

$$C_0(u,v) = \hat{u} * C\left[\lfloor un \rfloor + 1, \lfloor vm \rfloor\right] + (1-\hat{u}) * C\left[\lfloor un \rfloor, \lfloor vm \rfloor\right]$$

$$C_1(u,v) = \hat{u} * C\left[\lfloor un \rfloor + 1, \lfloor vm \rfloor + 1\right] + (1-\hat{u}) * C\left[\lfloor un \rfloor, \lfloor vm \rfloor + 1\right]$$

$$C_{tex}(u,v) = \hat{v} * C_1(u,v) + (1-\hat{v}) * C_0(u,v)$$

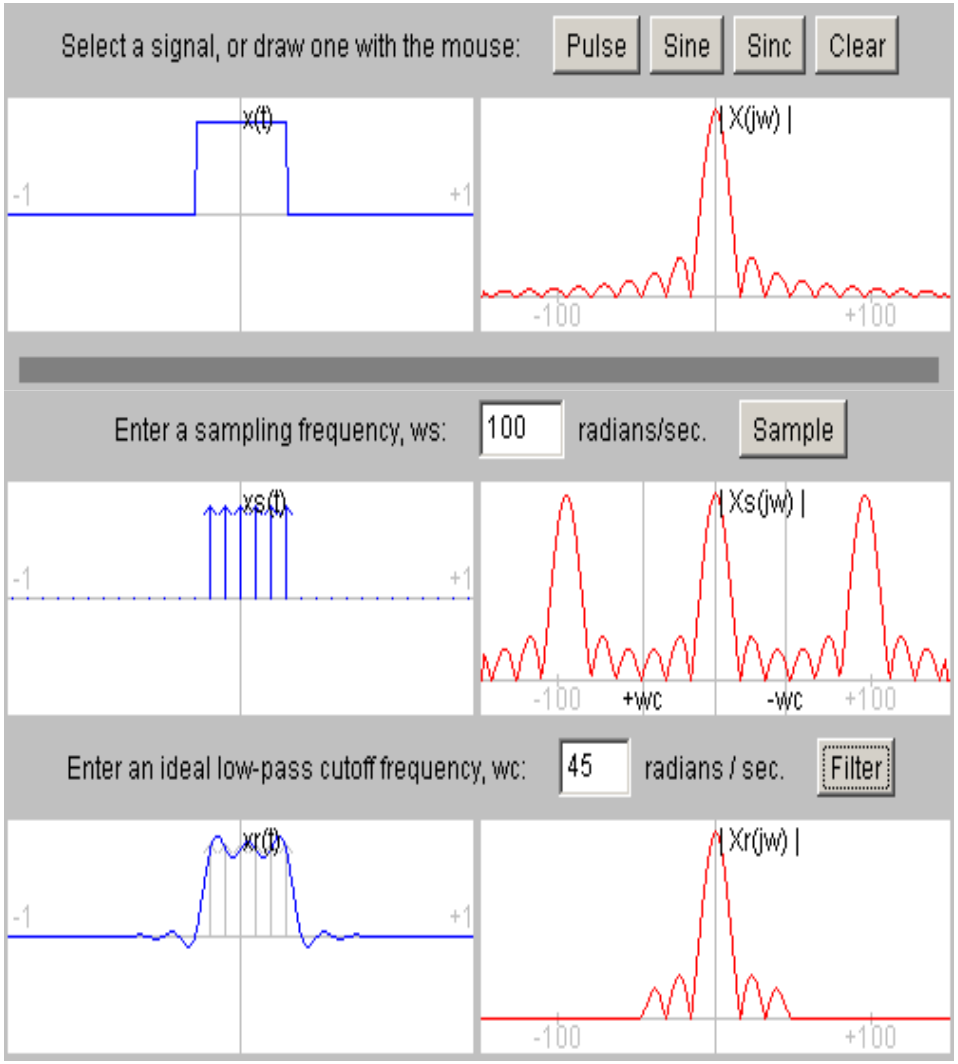
Discrete textures

Reconstruction

- Bilinear interpolation is a good approximation
- Better: convolution with a sinc filter

$$\operatorname{sinc}(x) \equiv \begin{cases} 1 & \text{for } x = 0 \\ \frac{\sin x}{x} & \text{otherwise,} \end{cases}$$

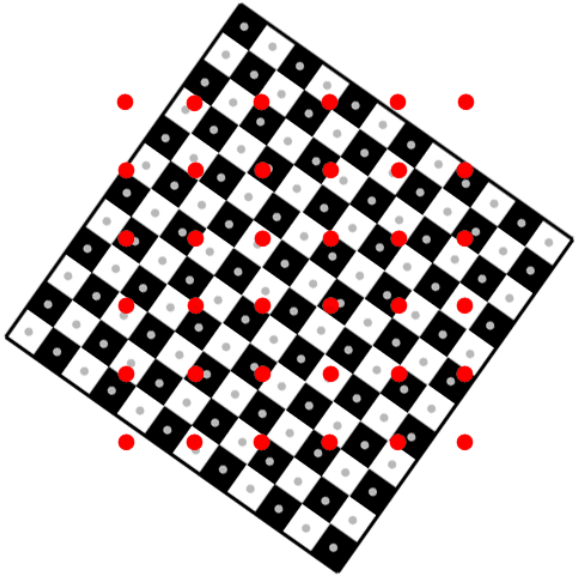
- See
  - http://www.jhu.edu/~signals/sampling/

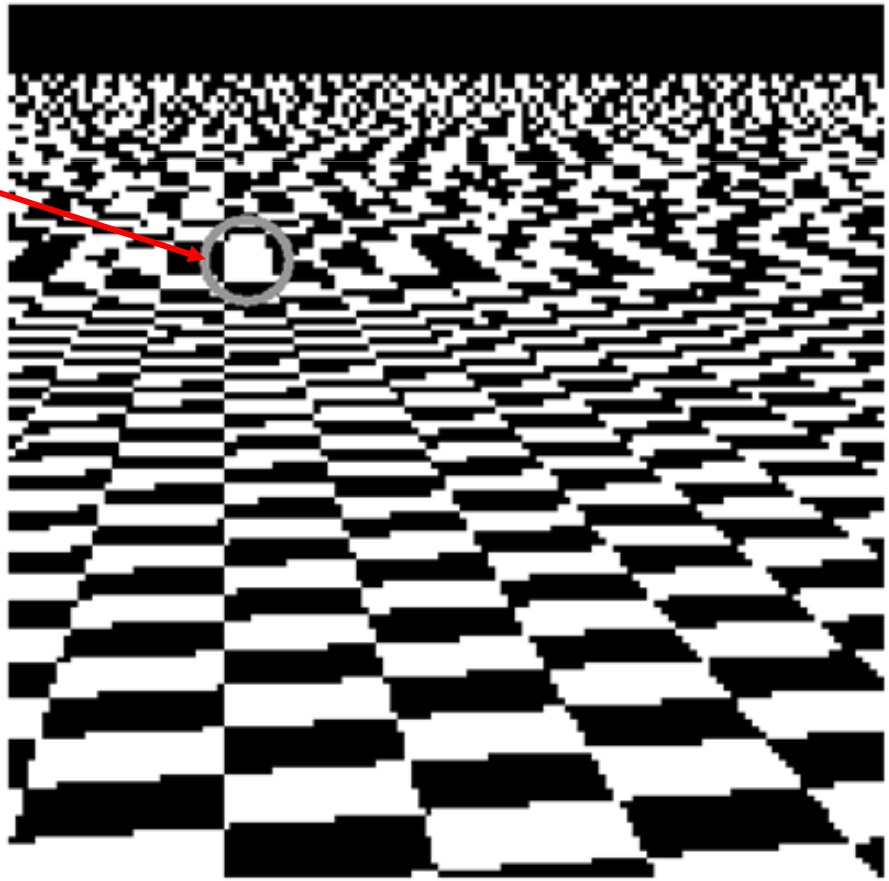


Discrete textures

Filtering

- Nearest neighbor filtering is insufficient for texture minification
- Undersampling errors create aliasing artifacts

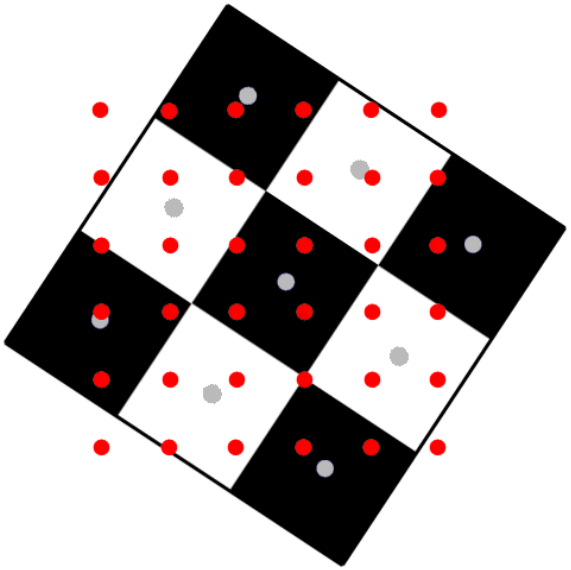


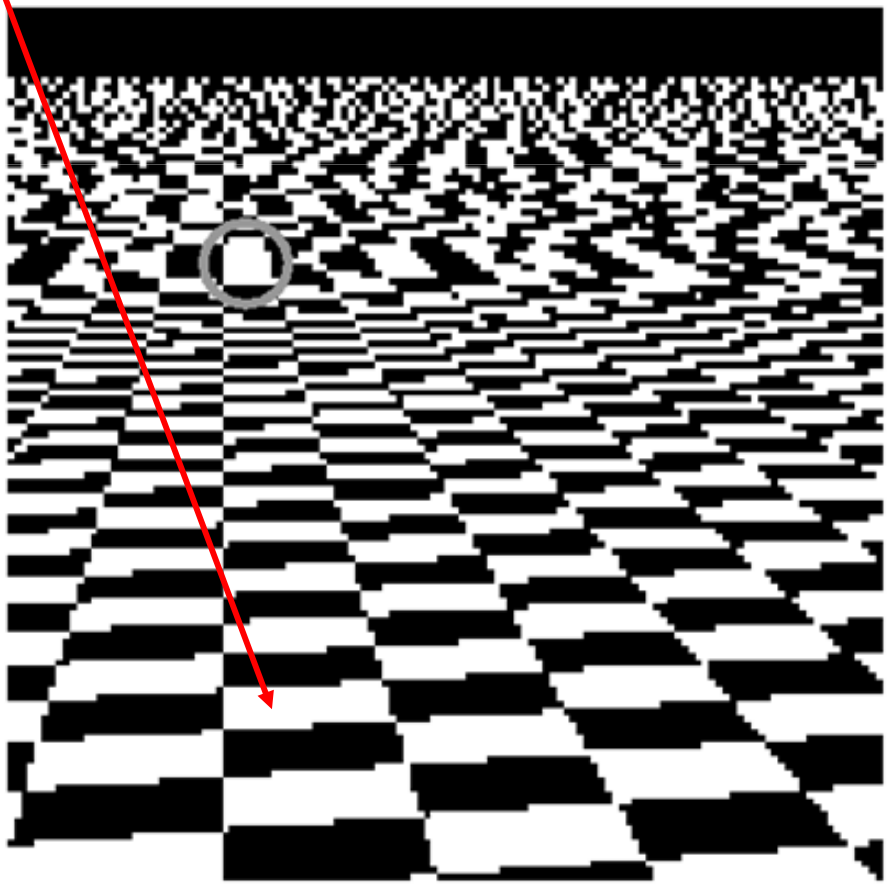


Discrete textures

Filtering

- Magnification (pixelation, blurring) is generally preferred over aliasing
- Achieved by reducing original texture res.

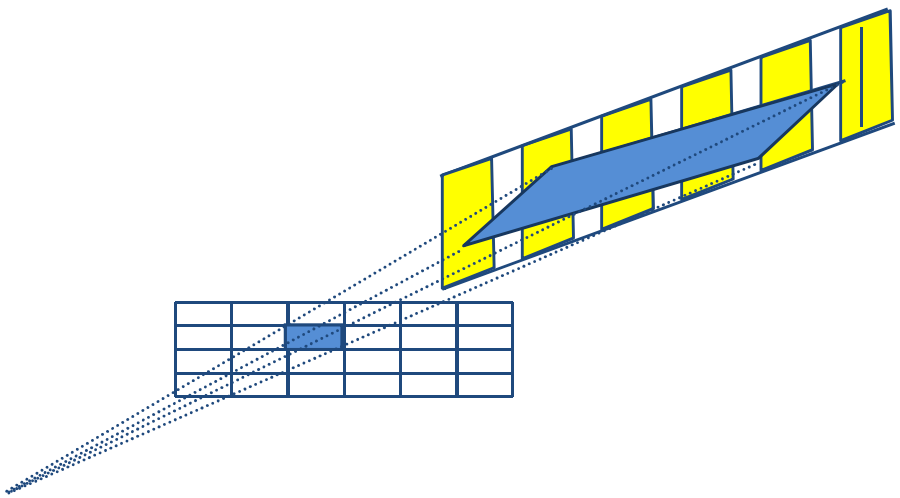


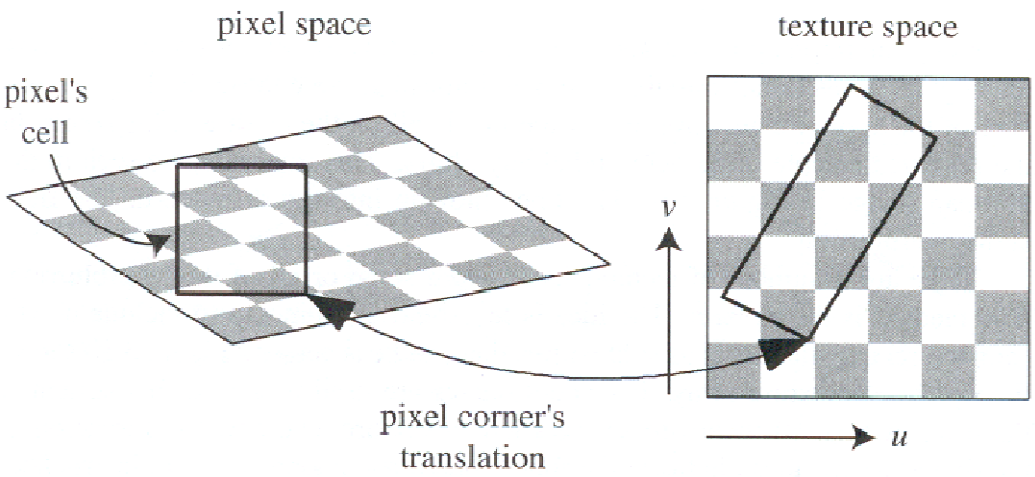


Discrete textures

Filtering

- **Footprint**: projection of the cell associated with a screen pixel (x,y) to texture (u,v) space
- Approximated by a parallelogram, spanned by the vectors $r_1 = (\frac{\partial u}{\partial x}, \frac{\partial v}{\partial x})^t,\ r_2 = (\frac{\partial u}{\partial y}, \frac{\partial v}{\partial y})^t$

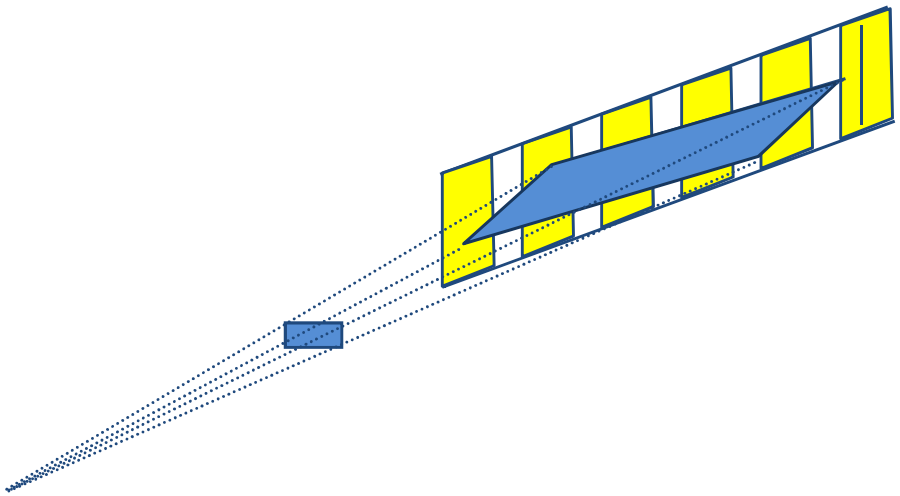


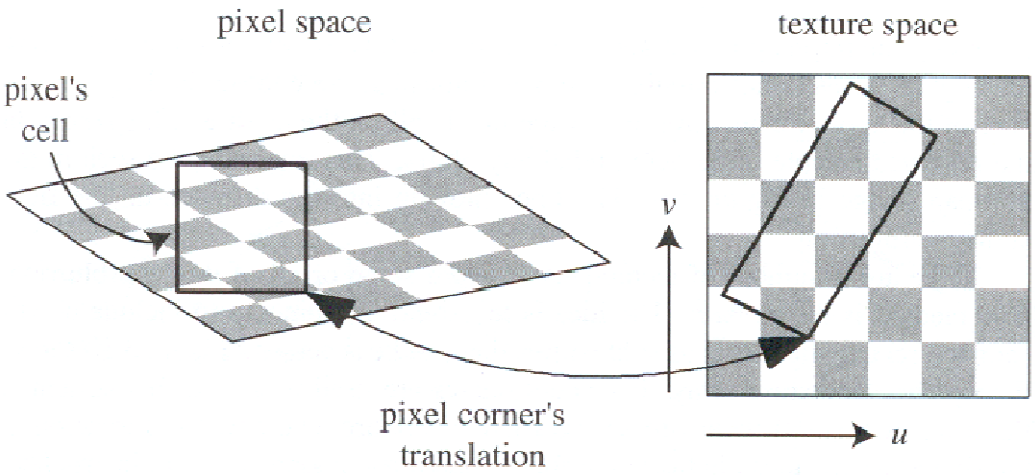


Discrete textures

Filtering

- For real-time texture filtering
  - Replace the actual footprint by a union of simpler surfaces
  - Such that the filtered texture value can be pre-computed

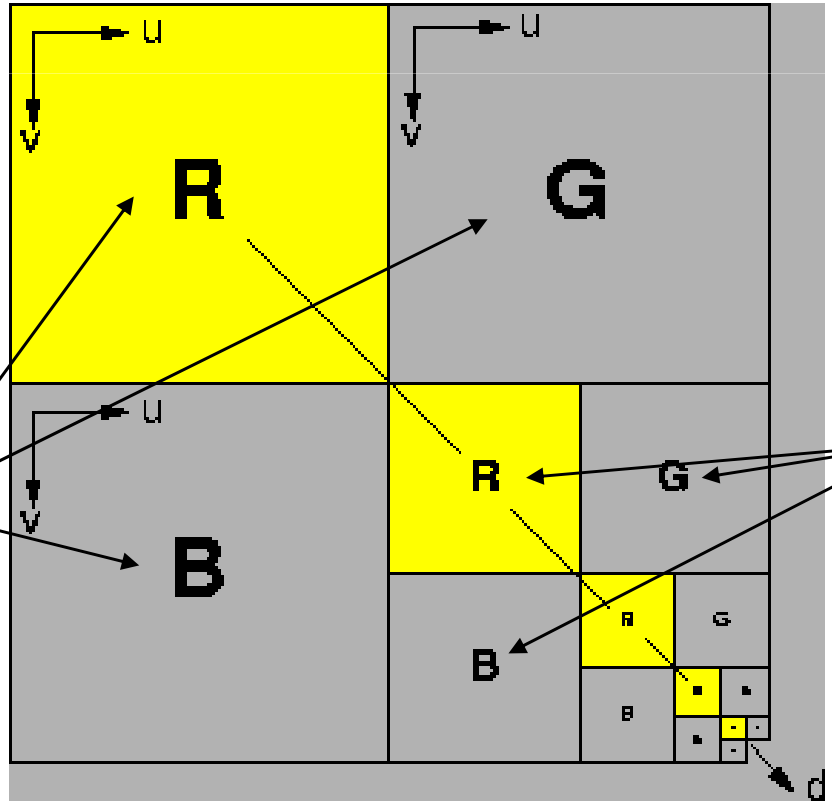




Discrete textures

Filtering: mip-map

- A 2n×2n mip-map C_{mip} [i,j] stores a 2D texture C[i,j] of size n×n where $n = 2^k$
  - sequentially at **half** the previous resolution



Color channels in full resolution

Next smaller resolution (filtered)

Discrete textures

Filtering: mip-map

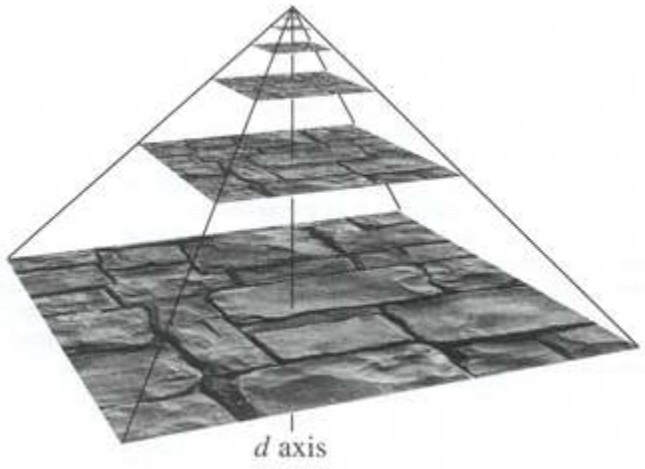
- Level d=0 are the original texture values

$$C^0_{mip}[i,j] = C[i,j],\ 0 \le \mathrm{i,j} < 2^{\mathrm{k}}.$$

- All other levels are constructed by filtering the previous resolution (weighted summation)

$$C^d_{mip}[i,j] = \frac{1}{4}\left(C^{d-1}_{mip}[2i,2j] + C^{d-1}_{mip}[2i+1,2j] + C^{d-1}_{mip}[2i,2j+1] + C^{d-1}_{mip}[2i+1,2j+1]\right)$$

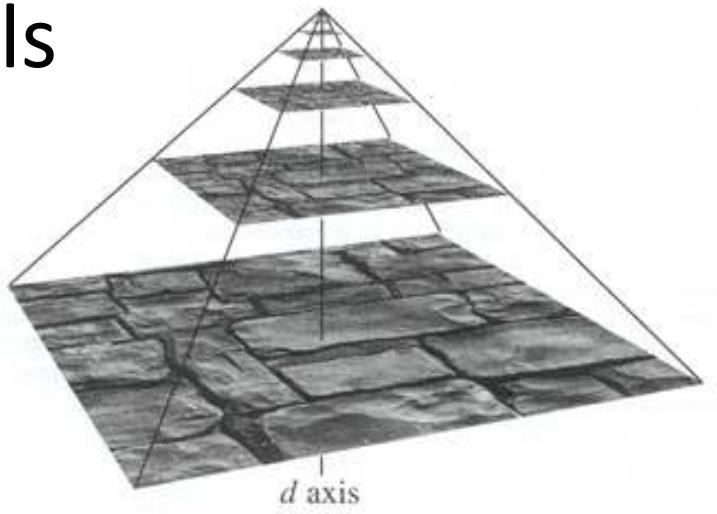
$$1 \le d < k\text{-}1 \text{ and } 0 \le i,j < 2^{k\text{-}d}.$$



Discrete textures

Filtering: mip-map

- On level d of the texture hierarchy, 2^{2d} texels of the original texture are represented as a single texel
- In the application, pixel values are interpolated from two levels depending on the size of the **footprint**
- This known as trilinear filtering



Discrete textures

Filtering: mip-map

- For a rectangular footprint, l=max(a,b)
- Too small footprints (e.g. (a+b)/2) can lead to aliasing (too large footprints to blurring)
- Texture value at level d= $\log_2(l)$ is

$$C_{tex}(u,v,d) = BiLinInt(C^d_{mip}, u, v)$$

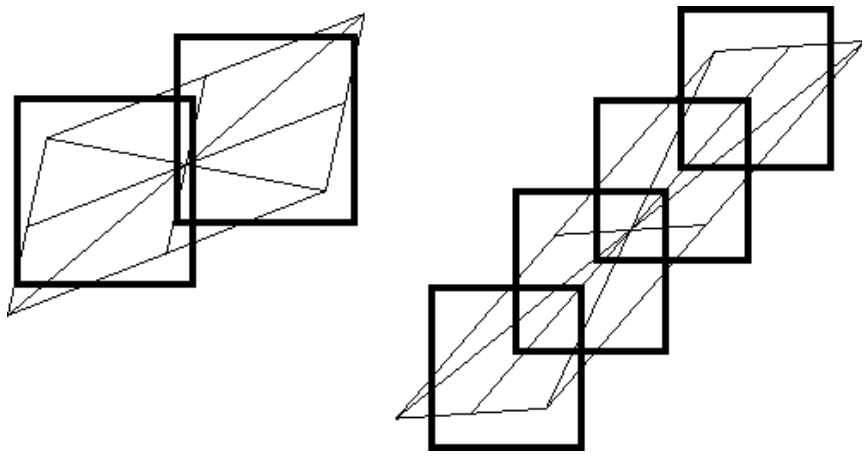
- When d is not an integer value
  - Interpolate between two levels $\lfloor d \rfloor$ und $\lceil d \rceil$

$$C_{tex}(u,v,d) = (d - \lfloor d \rfloor) * BiLinInt(C^{\lfloor d \rfloor + 1}_{mip}, u, v) + (\lfloor d \rfloor + 1 - d) * BiLinInt(C^{\lfloor d \rfloor}_{mip}, u, v).$$

Discrete textures

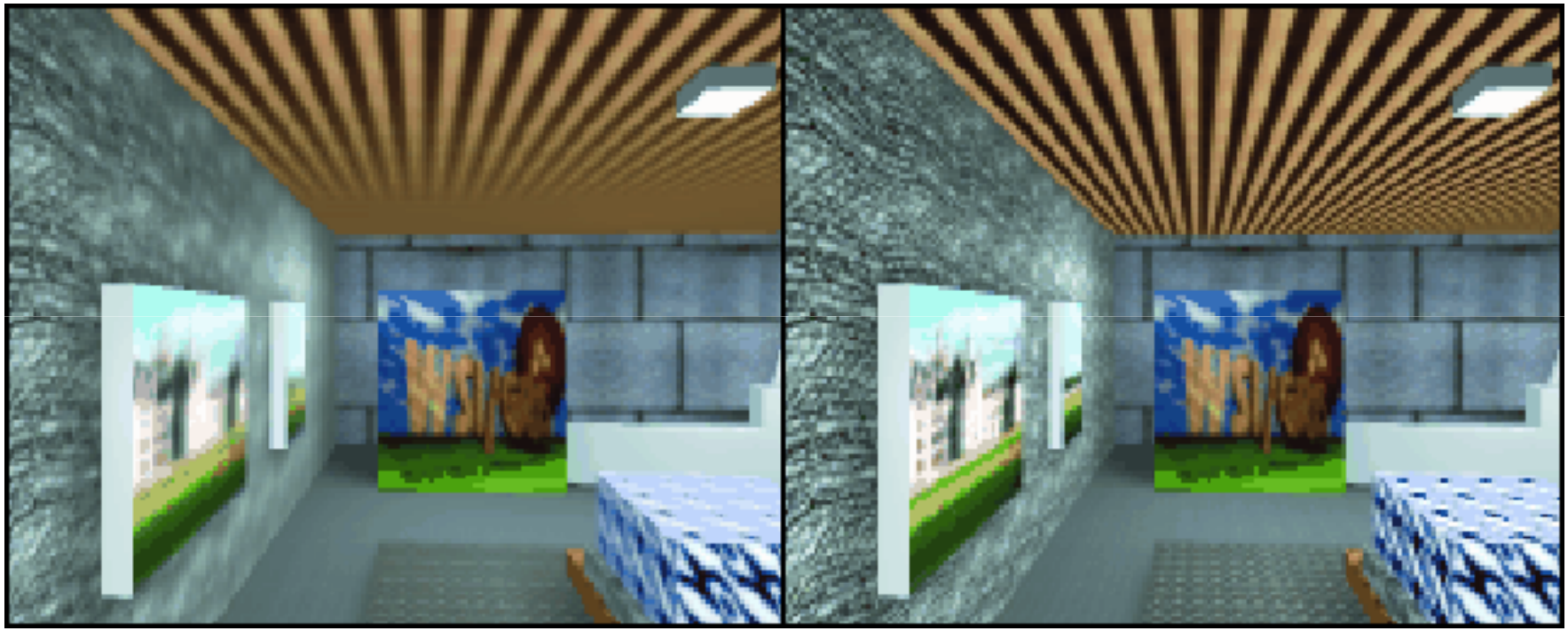
Filtering: footprint assembly

- Square projection ignores anisotropy!
- Idea: use multiple mip-map accesses to cover the footprint parallelogram
- **Anisotropic texture filtering** in real time
  - Number of square parts determines the quality of the anisotropic filter



Discrete textures

Filtering: footprint assembly



Mip-mapping

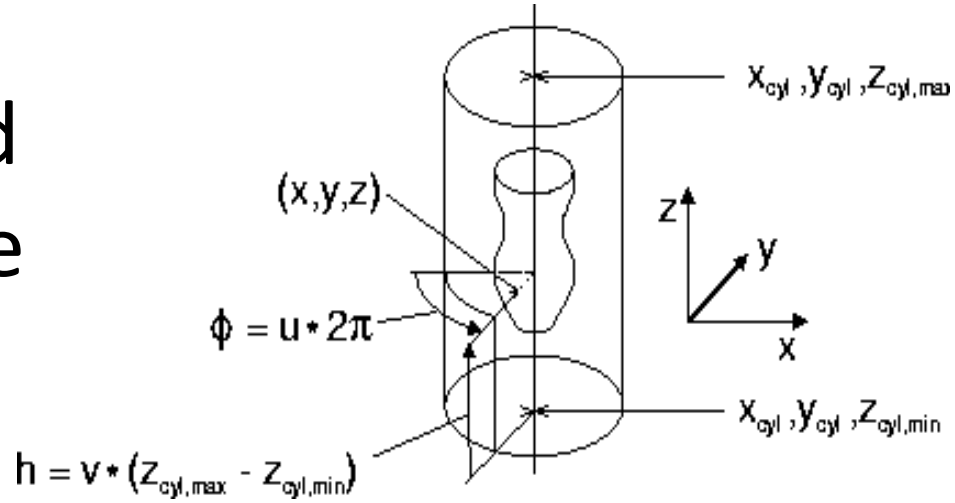
Anisotropic filtering

Texturing surfaces

- How to determine the texture coordinates for the vertices of a polygonal model?
  - They might be given by a modeler ☺
  - If not, embed the complex object in a simpler object for which a simple (u,v) parameterization exists
  - Project from the polygonal model onto the simpler object
  - Examples: cylinder, sphere, box

Cylinder mapping

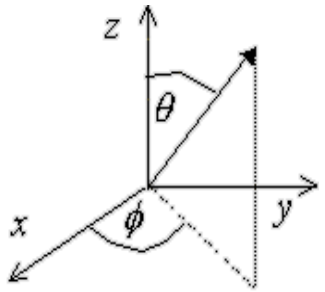
- Parameterizaton:
  rotation angle ϕ and height h
- Points interior to the cylinder are projected orthogonally from the axis to the cylinder surface



  - Interpret (ϕ ,h) parameters as (u,v) texture coordinates

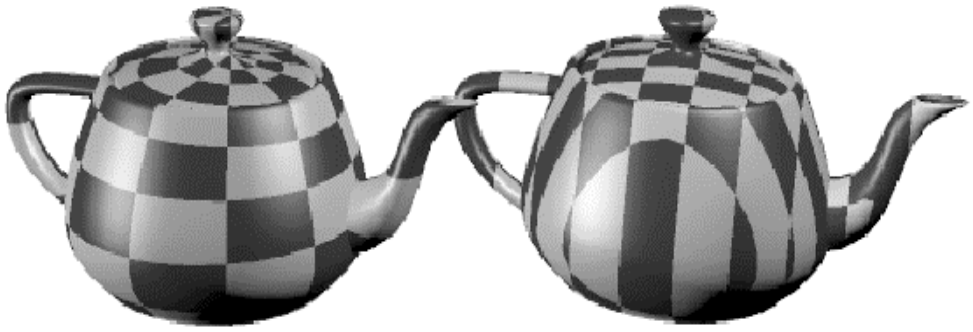
Sphere mapping

- Polar coordinates ϕ and θ
- Project points (x,y,z) on the interior through the midpoint (x_s,y_s,z_s) onto the sphere
- Interpret (ϕ,θ) as (u,v)



$$u = \frac{\pi + \arctan2\left(y - y_s, x - x_s\right)}{2\pi}$$

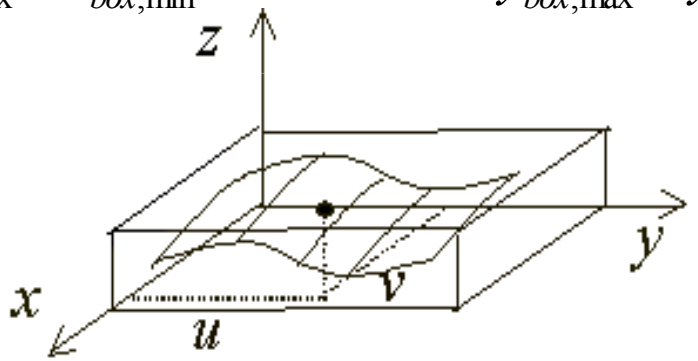
$$v = \frac{\arctan2\left(\sqrt{(x-x_s)^2 + (y-y_s)^2}, z - z_s\right)}{\pi}$$



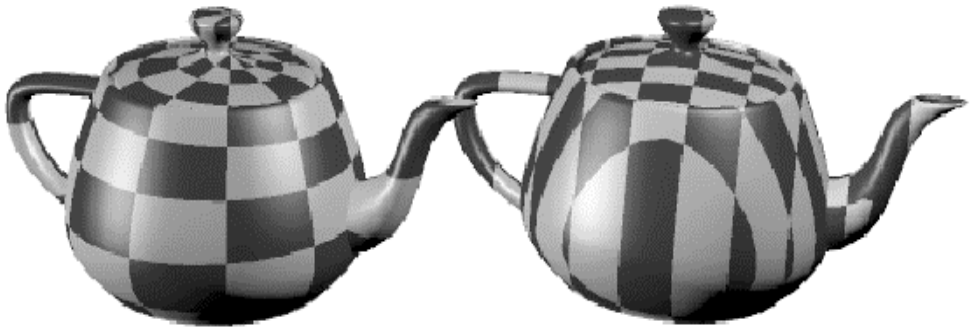
Box mapping

$$(x_{box,\max} - x_{box,\min}) \geq (y_{box,\max} - y_{box,\min}) \geq (z_{box,\max} - z_{box,\min})$$

$$u = \frac{x - x_{box,\min}}{x_{box,\max} - x_{box,\min}} \qquad v = \frac{y - y_{box,\min}}{y_{box,\max} - y_{box,\min}}$$



Sphere mapping



Box mapping