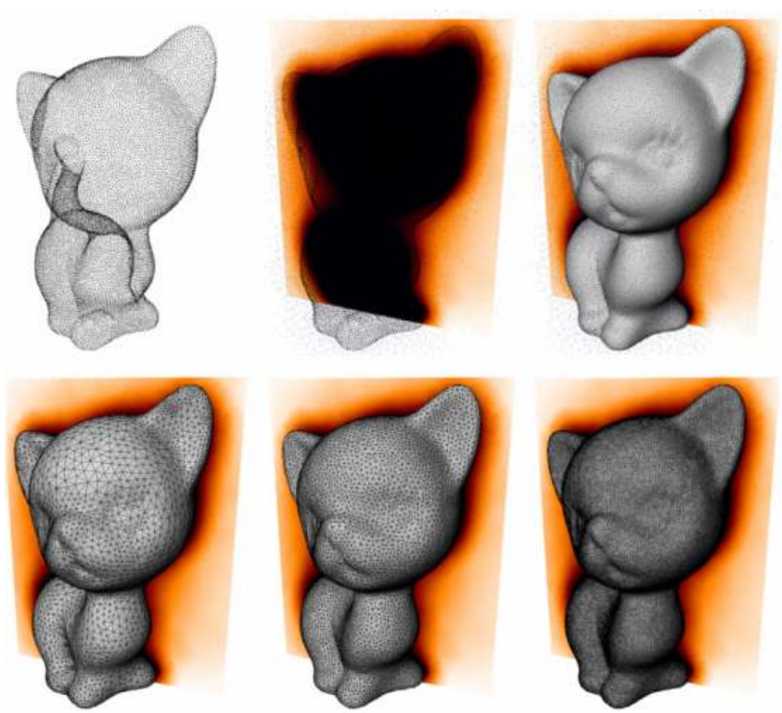
CS 523: Computer Graphics, Spring 2011

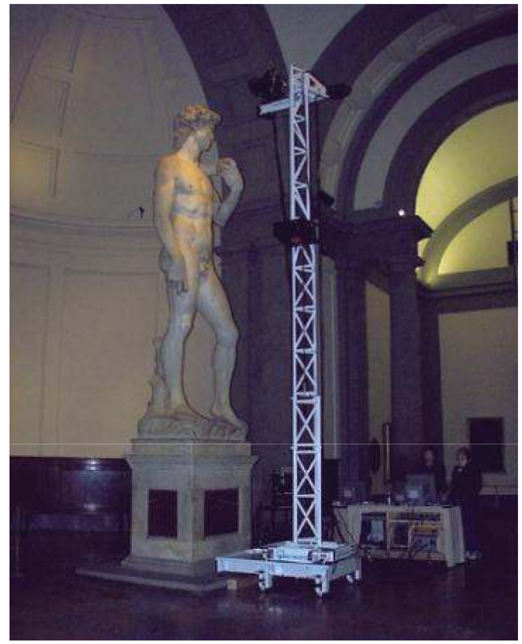
Shape Modeling

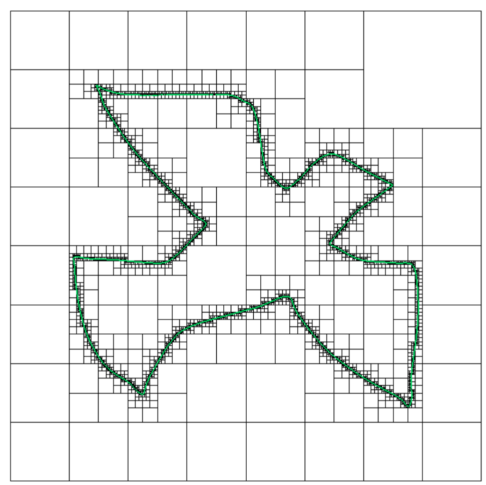
Shape Reconstruction

Course Topics

- Shape acquisition
  - Scanning/imaging
  - Reconstruction







Data Acquisition

Pipeline

Scanning: results in range images → **Registration:** bring all range images to one coordinate system → **Stitching/reconstruction:** Integration of scans into a single mesh → **Postprocess:**

- Topological and geometric filtering
- Remeshing
- Compression

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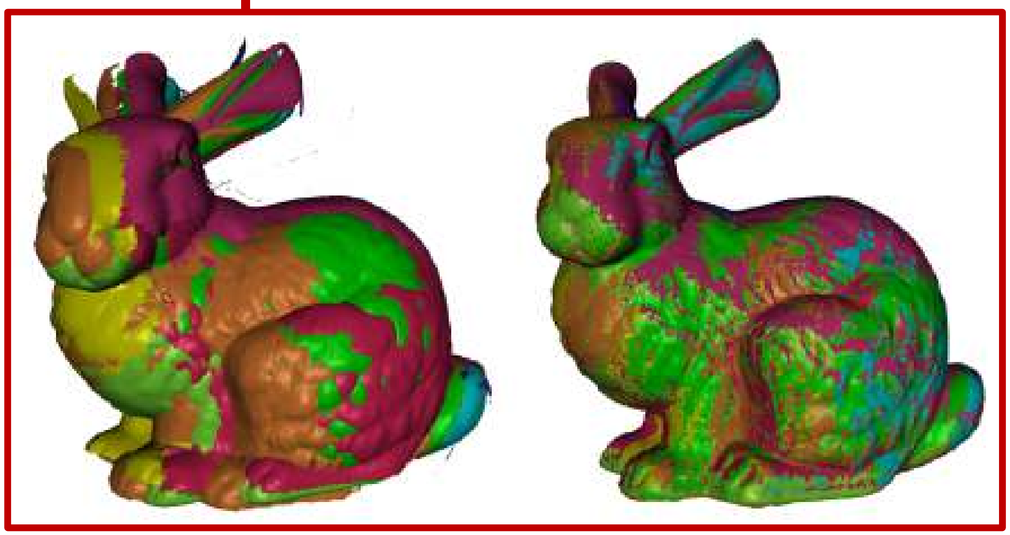

Data Acquisition
Pipeline

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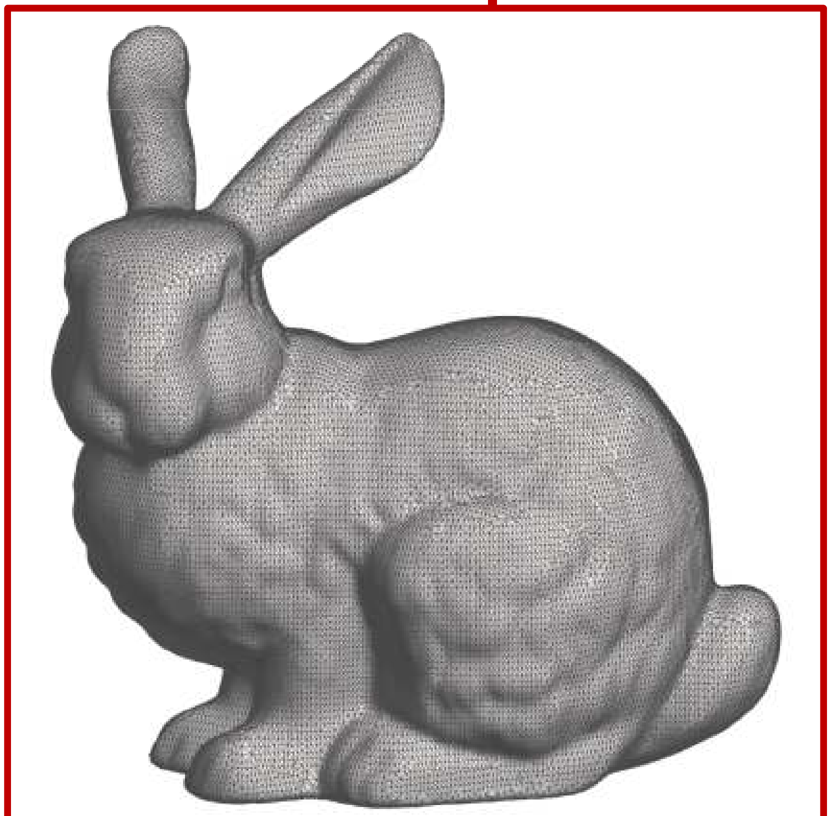

Data Acquisition

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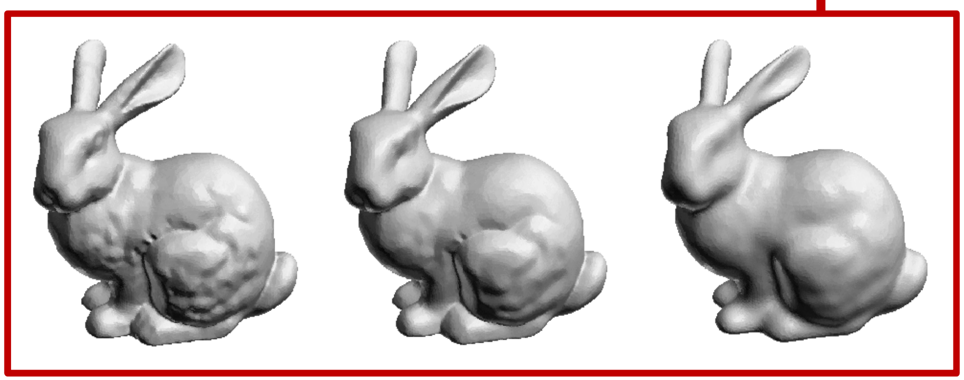
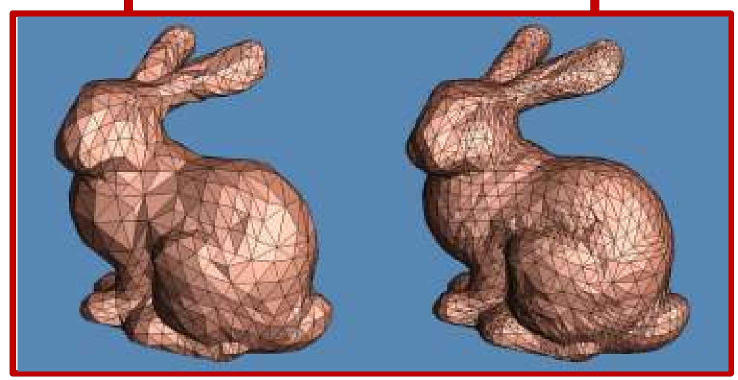




Touch probes

- Physical contact with the object
- Manual or computer-guided
- Advantages:
  - Can be very precise
  - Can scan any solid surface
- Disadvantages:
  - Slow, small scale
  - Can't use on fragile objects

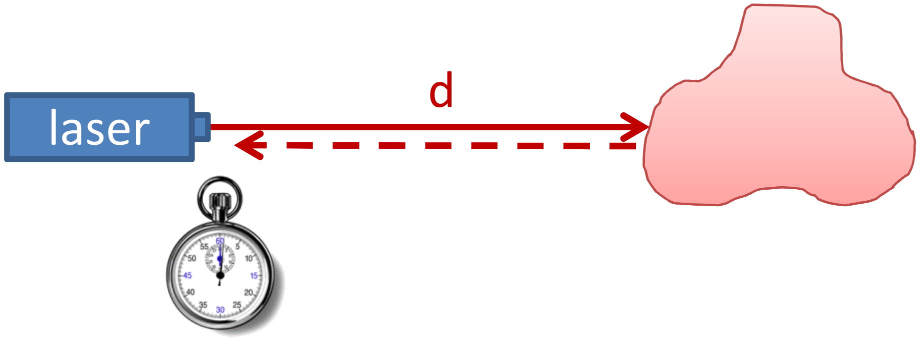
Optical scanning

- Infer the geometry from light reflectance
- Advantages:
  - Less invasive than touch
  - Fast, large scale possible
- Disadvantages:
  - Difficulty with transparent and shiny objects

Optical scanning – active lighting

Time of flight laser

- Laser rangefinder (lidar)
- Measures the time it takes the laser beam to hit the object and come back
- Scans one point at a time; mirrors used to change beam direction

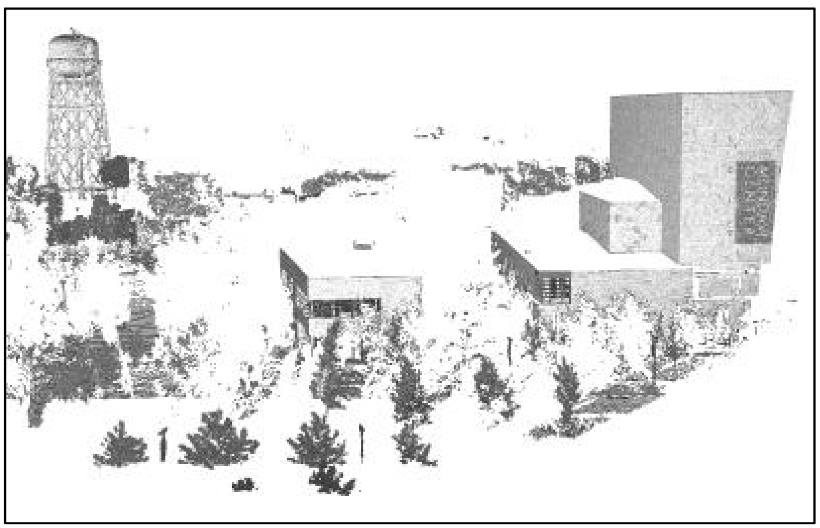


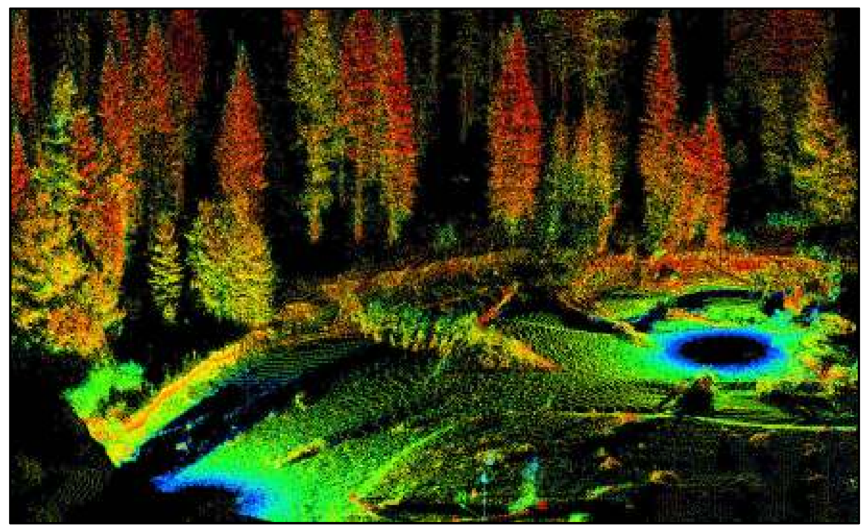
$d = 0.5\, t \cdot c$

Optical scanning – active lighting

Time of flight laser

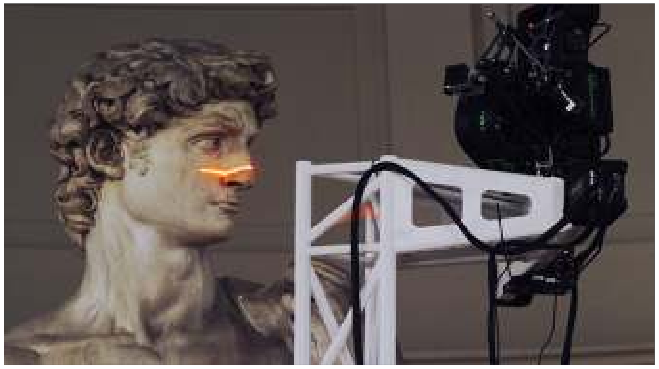
- Accommodates large range – up to several miles (suitable for buildings, rocks)
- Lower accuracy (light travels really fast)



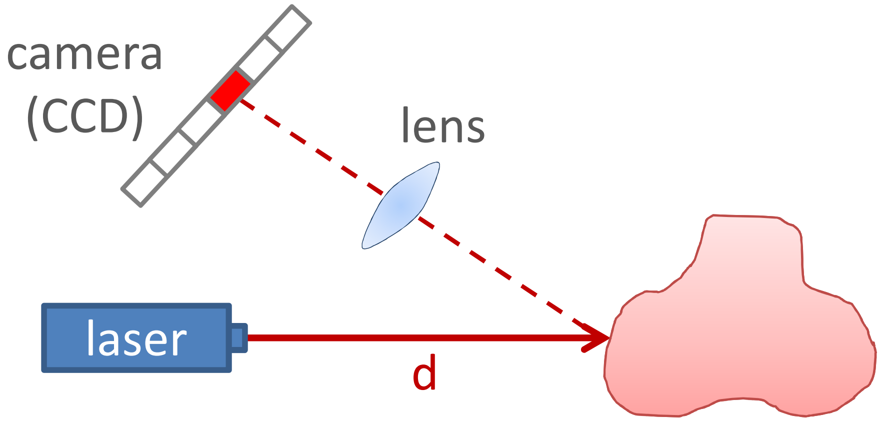


Optical scanning – active lighting

Triangulation laser

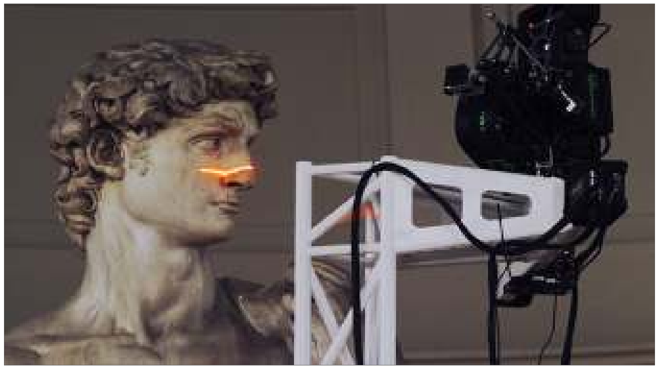


- Laser beam and camera
- Laser dot is photographed
- The location of the dot in the image allows triangulation – so we get the distance to the object



Optical scanning – active lighting

Triangulation laser



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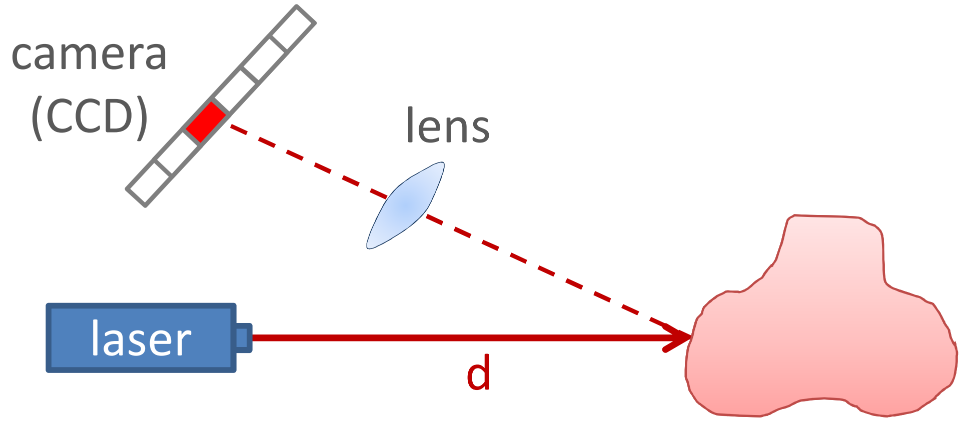

Optical scanning – active lighting

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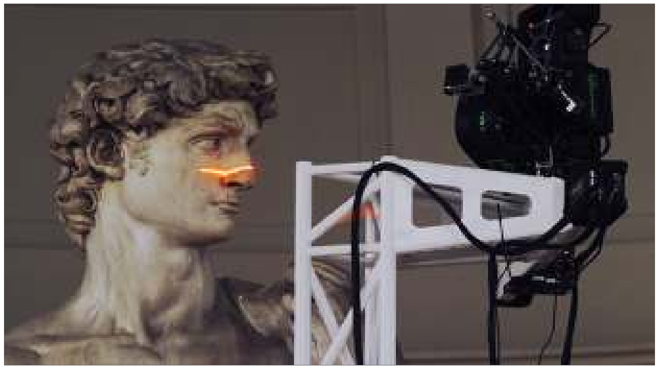
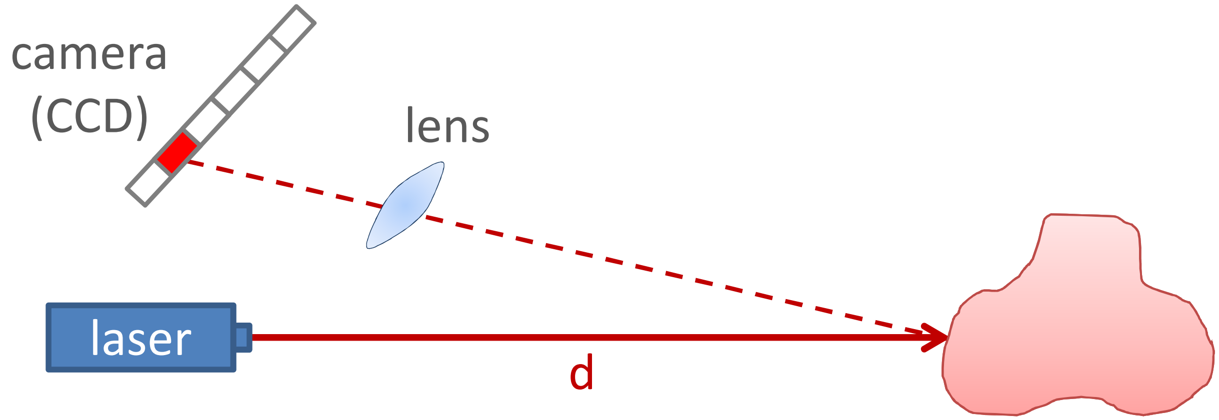
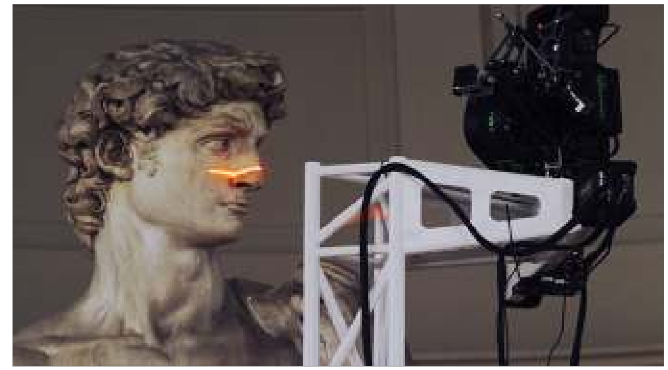




Optical scanning – active lighting

Triangulation laser

- Speed-up: instead of a single dot, a whole stripe is swiped across the object



- Very precise (tens of microns)
- Small distances (meters)

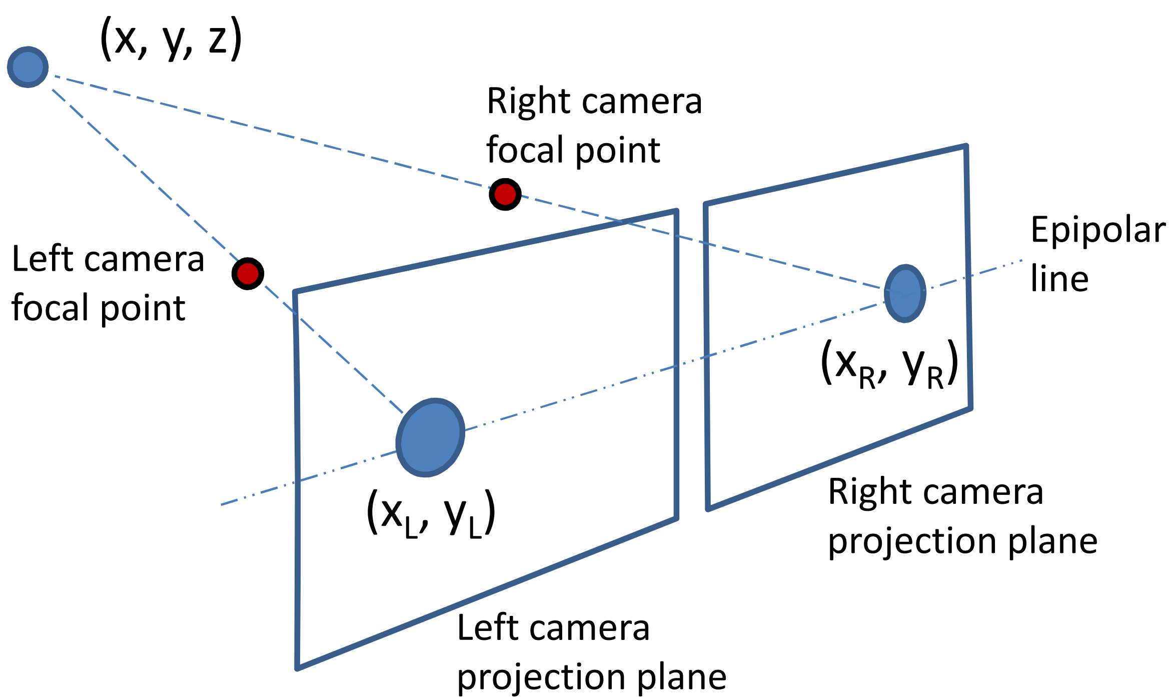
Optical scanning – active lighting
Structured light

- Pattern of visible light is projected onto the object
- The distortion of the pattern, recorded by the camera, provides geometric information
- Very fast – 2D pattern at once, not single dots/lines
  - Even in real time
- Complex distance calculation, prone to noise

Optical scanning – passive

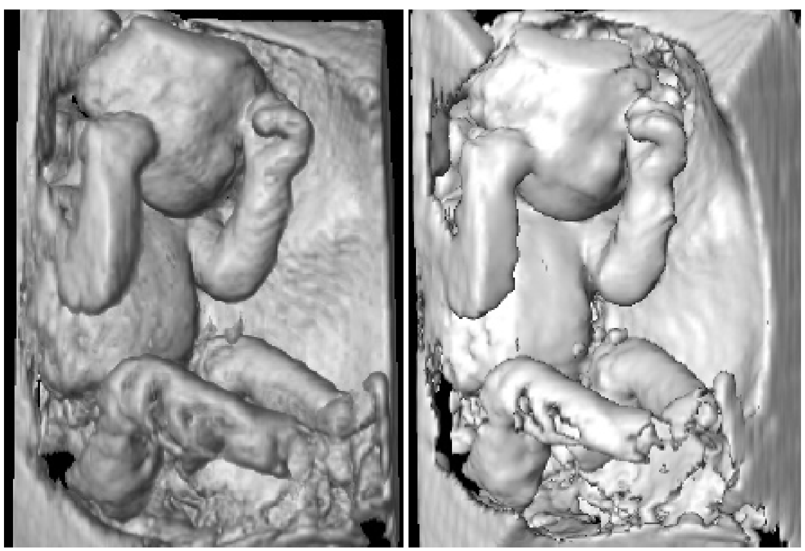
Stereo

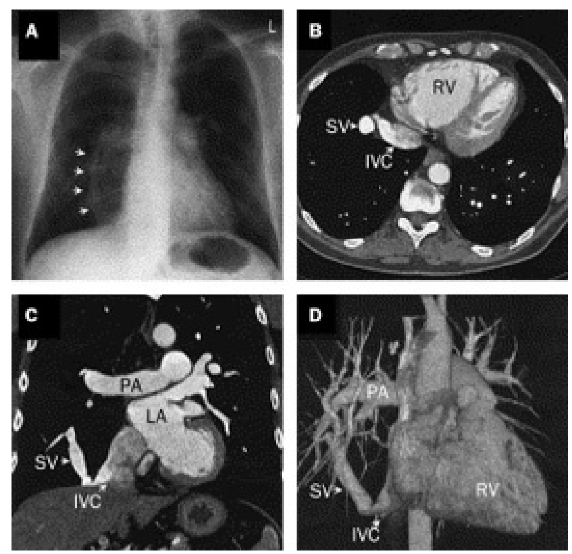
- No need for special lighting/radiation
- Two (or more) cameras
- Feature matching and triangulation



Imaging

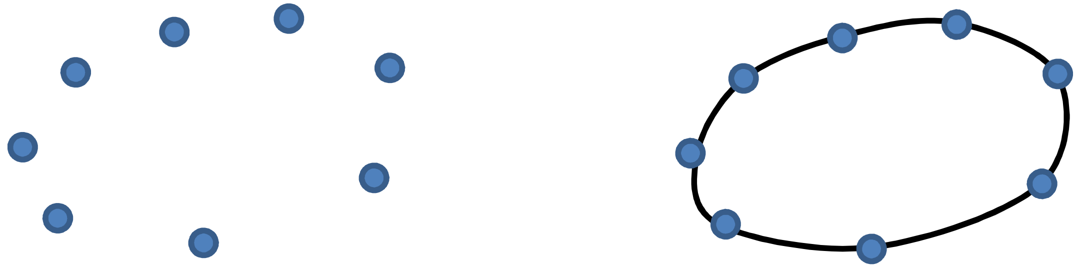
- Ultrasound, CT, MRI
- Discrete volume of density data
- First need to segment the desired object (contouring)





Surface reconstruction

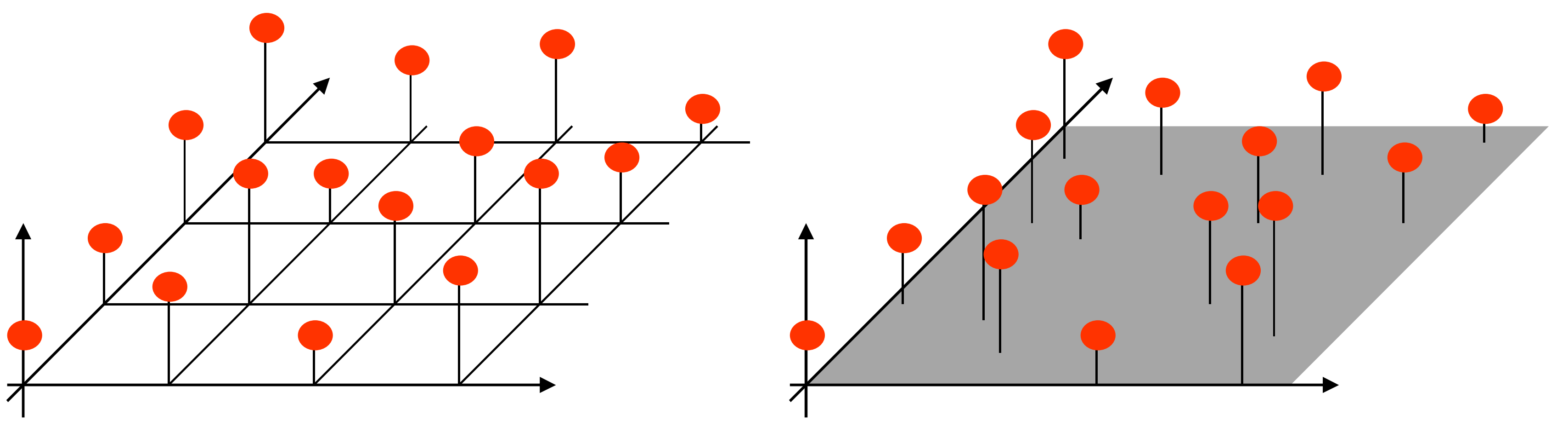
- How to create a single mesh?
  - Surface topology?
  - Smoothness?
  - How to connect the dots?



Continuous reconstruction

2D Example

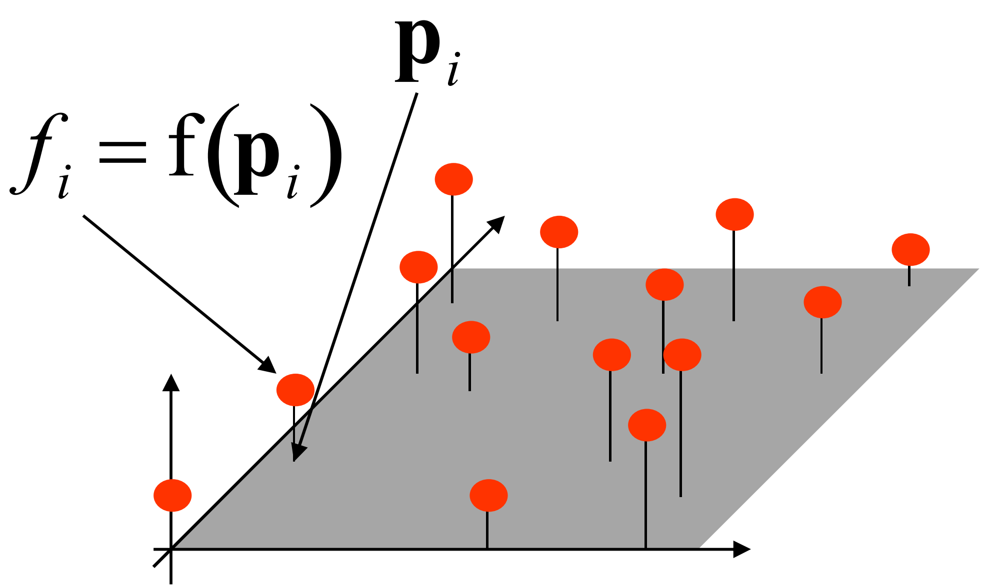
- Given a set of scattered (scalar) data points f_i at positions $\mathbf{p}_i$ in a 2D parameter domain
- The principles are applicable to arbitrary parameter domain dimensions



Continuous reconstruction

2D Example

- The reconstruction operates on a single dimension (i.e. the z-component) of the parametric (hyper) surface
- Goal: approximate function f from $f_i, \mathbf{p}_i$

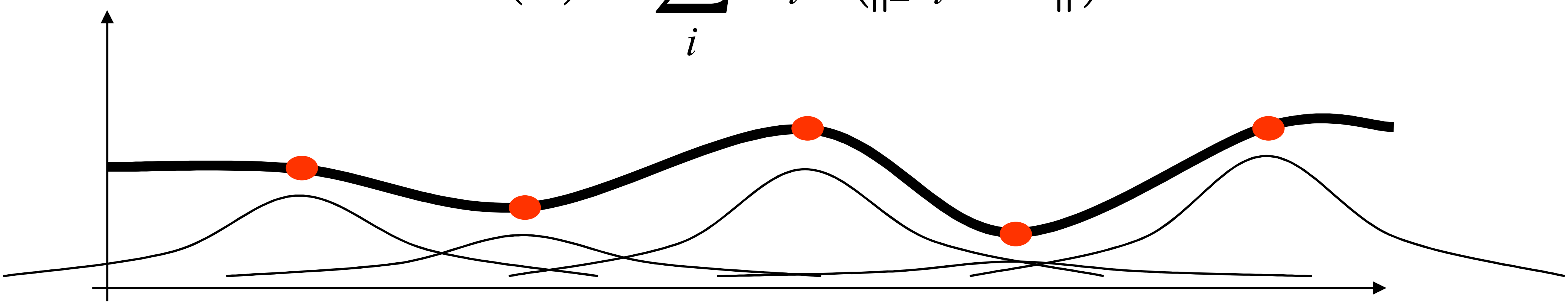


Radial Basis Functions

1D Example

- Independent of parameter domain dimension
- Function f represented as
  - Weighted sum of radial functions r
  - In the parameter domain positions $\mathbf{p}_{\mathrm{i}}$

$$\mathrm{f}(\mathbf{x}) = \sum_i w_i \, \mathrm{r}\left(\left\|\mathbf{p}_i - \mathbf{x}\right\|\right)$$



Radial Basis Functions

Computing the coefficients

- Set

$$f_j = \sum_i w_i \, \mathrm{r}\left(\left\| t_i - t_j \right\|\right)$$

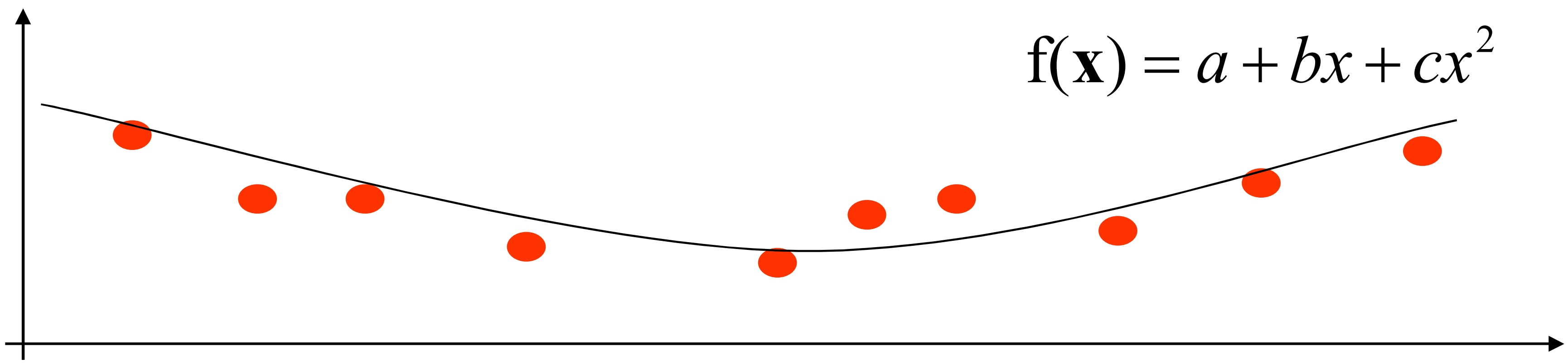
  to compute the weights/coefficients w_i

- Linear system of equations (per dimension)

$$\begin{pmatrix} \mathrm{r}(0) & \mathrm{r}\left(\left\|t_0 - t_1\right\|\right) & \mathrm{r}\left(\left\|t_0 - t_2\right\|\right) & \cdots \\ \mathrm{r}\left(\left\|t_1 - t_0\right\|\right) & \mathrm{r}(0) & \mathrm{r}\left(\left\|t_1 - t_2\right\|\right) & \\ \mathrm{r}\left(\left\|t_2 - t_0\right\|\right) & \mathrm{r}\left(\left\|t_2 - t_1\right\|\right) & \mathrm{r}(0) & \\ \vdots & & & \ddots \end{pmatrix} \begin{pmatrix} w_0 \\ w_1 \\ w_2 \\ \vdots \end{pmatrix} = \begin{pmatrix} f_0 \\ f_1 \\ f_2 \\ \vdots \end{pmatrix}$$

Global Approximation

- Given $\mathbf{p}_i \in \mathbb{R}^d,\ f_i \in \mathbb{R},\ i = 0,\ldots,n$
  - $\mathbf{p}_i$ - parameter domain positions
  - f_i - function values
- Compute polynomial curve $\mathrm{f}(\mathbf{p}_i) \approx f_i,\ i = 0,\ldots,n$



Least Squares Approximation

- Error functional

$$J_{LS} = \sum_i \|f(\mathbf{x}_i) - f_i\|^2$$

- Polynomial basis of degree m in d dimensions

$$f(\mathbf{x}) = \mathbf{b}(\mathbf{x})^T \mathbf{c} = \mathbf{b}(\mathbf{x}) \cdot \mathbf{c}$$

$$\mathbf{b}(\mathbf{x}) = [b_1(\mathbf{x}), \ldots, b_k(\mathbf{x})]^T \qquad \mathbf{c} = [c_1, \ldots, c_k]^T$$

$$\mathbf{b}(\mathbf{x}) = [1, x, y, x^2, xy, y^2]^T$$

- Previous 1D quadratic Example $f(\mathbf{x}) = c_1 + c_2 x + c_3 x^2$

Least Squares Approximation

- Solve for $\mathbf{c}$ by taking (partial) derivatives of J_{LS} w.r.t. the unknowns and setting to zero

$$
\begin{aligned}
\partial J_{LS}/\partial c_1 = 0: &\qquad \sum_i 2b_1(\mathbf{x}_i)[\mathbf{b}(\mathbf{x}_i)^T\mathbf{c} - f_i] = 0 \\
\partial J_{LS}/\partial c_2 = 0: &\qquad \sum_i 2b_2(\mathbf{x}_i)[\mathbf{b}(\mathbf{x}_i)^T\mathbf{c} - f_i] = 0 \\
&\vdots \\
\partial J_{LS}/\partial c_k = 0: &\qquad \sum_i 2b_k(\mathbf{x}_i)[\mathbf{b}(\mathbf{x}_i)^T\mathbf{c} - f_i] = 0.
\end{aligned}
$$

Least Squares Approximation

- In matrix-vector notation

$$\sum_i 2\mathbf{b}(\mathbf{x}_i)[\mathbf{b}(\mathbf{x}_i)^T\mathbf{c} - f_i] \quad =$$

$$2\sum_i [\mathbf{b}(\mathbf{x}_i)\mathbf{b}(\mathbf{x}_i)^T\mathbf{c} - \mathbf{b}(\mathbf{x}_i)f_i] \quad = \quad \mathbf{0}.$$

$$\sum_i \mathbf{b}(\mathbf{x}_i)\mathbf{b}(\mathbf{x}_i)^T\mathbf{c} = \sum_i \mathbf{b}(\mathbf{x}_i)f_i$$

- Solve for $\mathbf{c} = [\sum_i \mathbf{b}(\mathbf{x}_i)\mathbf{b}(\mathbf{x}_i)^T]^{-1}\sum_i \mathbf{b}(\mathbf{x}_i)f_i$

Least Squares Approximation

2D quadratic example

- Error functional and partial derivatives

$$f(\mathbf{x}) = a + b_u u + b_v v + c_{uu} u^2 + c_{uv} uv + c_{vv} v^2$$

$$\min_{(a,\mathbf{b},\mathbf{C})} \sum_i \left(\mathrm{f}(u_i, v_i) - f_i\right)^2 = \min_{(a,\mathbf{b},\mathbf{C})} \sum_i \left(a + b_u u_i + b_v v_i + c_{uu} u_i^2 + c_{uv} u_i v_i + c_{vv} v_i^2 - f_i\right)^2$$

$$\partial \sum_i \left(\mathrm{f}(u_i, v_i) - f_i\right)^2 \Big/ \partial a = \sum_i 2\left(a + b_u u_i + b_v v_i + c_{uu} u_i^2 + c_{uv} u_i v_i + c_{vv} v_i^2 - f_i\right) = 0$$

$$\vdots$$

$$\partial \sum_i \left(\mathrm{f}(u_i, v_i) - f_i\right)^2 \Big/ \partial c_{vv} = \sum_i 2 v_i^2 \left(a + b_u u_i + b_v v_i + c_{uu} u_i^2 + c_{uv} u_i v_i + c_{vv} v_i^2 - f_i\right) = 0$$

Least Squares Approximation

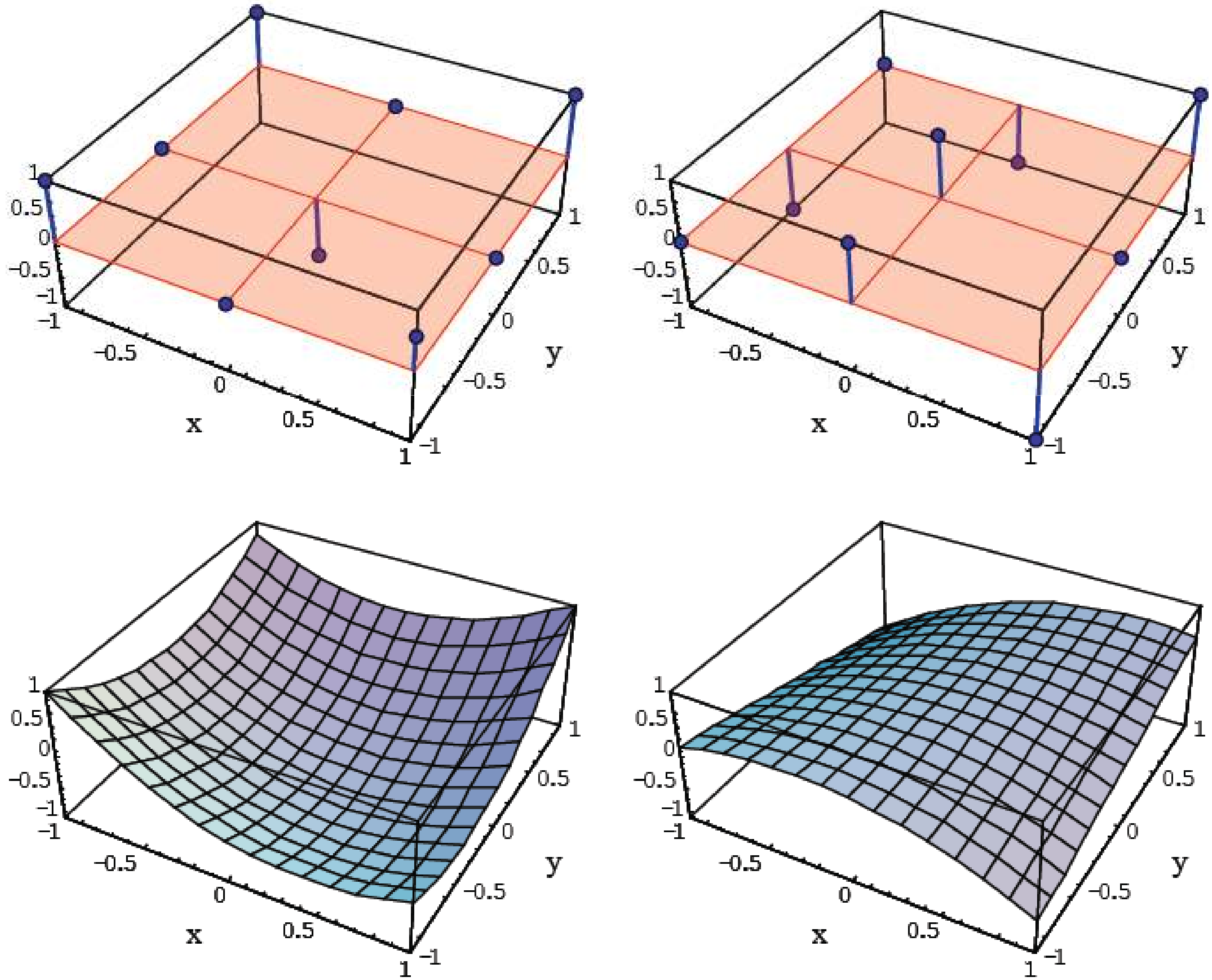
2D quadratic example

- Linear system of equations

$$\sum_i \begin{pmatrix} 1 & u_i & v_i & u_i^2 & u_i v_i & v_i^2 \\ u_i & u_i^2 & u_i v_i & u_i^3 & u_i^2 v_i & u_i v_i^2 \\ v_i & u_i v_i & v_i^2 & u_i^2 v_i & u_i v_i^2 & v_i^3 \\ u_i^2 & u_i^3 & u_i^2 v_i & u_i^4 & u_i^3 v_i & v_i^2 u_i^2 \\ u_i v_i & u_i^2 v_i & u_i v_i^2 & u_i^3 v_i & u_i^2 v_i^2 & u_i v_i^3 \\ v_i^2 & v_i^2 u_i & v_i^3 & v_i^2 u_i^2 & u_i v_i^3 & v_i^4 \end{pmatrix} \begin{pmatrix} a \\ b_u \\ b_v \\ c_{uu} \\ c_{uv} \\ c_{vv} \end{pmatrix} = \sum_i f_i \begin{pmatrix} 1 \\ u_i \\ v_i \\ u_i^2 \\ u_i v_i \\ v_i^2 \end{pmatrix}$$

Least Squares Approximation

Results



Least Squares Approximation

Normal equations

Method of Normal Equations. For a different but also very common notation, note that the solution for $\mathbf{c}$ solves the following (generally over-constrained) LSE ($\mathbf{Bc} = \mathbf{f}$) in the least-squares sense

$$\begin{bmatrix} \mathbf{b}^T(\mathbf{x}_1) \\ \vdots \\ \mathbf{b}^T(\mathbf{x}_N) \end{bmatrix} \mathbf{c} = \begin{bmatrix} f_1 \\ \vdots \\ f_N \end{bmatrix},$$

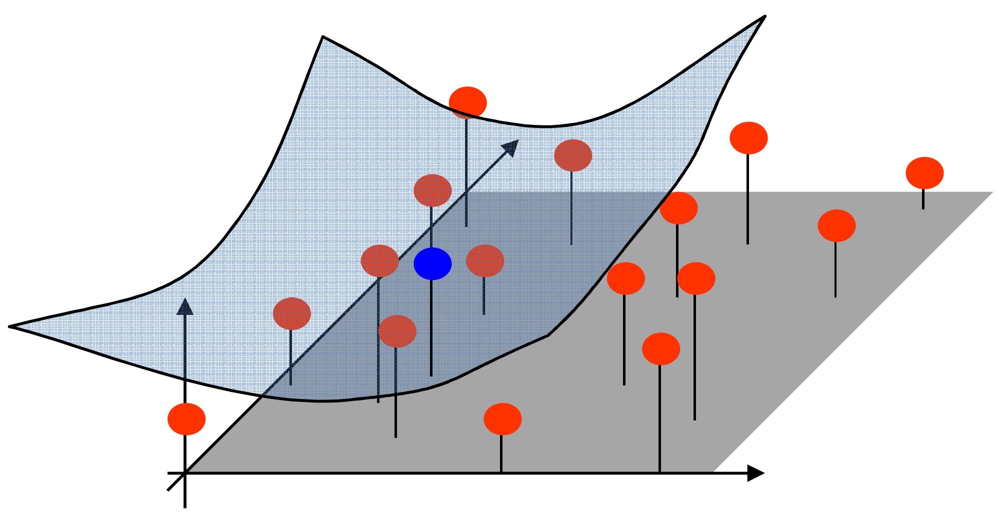
using the method of normal equations

$$\begin{aligned} \mathbf{B}^T\mathbf{B}\mathbf{c} &= \mathbf{B}^T\mathbf{f} \\ \mathbf{c} &= (\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T\mathbf{f}. \end{aligned}$$

Weighted Least Squares

- Principle: local approximation at $\bar{\mathbf{x}}$ by weighting the squared errors based on proximity in the parameter domain

$$\min_{\mathrm{f}_{\mathbf{x}} \in \Pi_k^d} \sum_{i=0}^{n} \left\| \mathrm{f}(\mathbf{p}_i) - f_i \right\|^2 \theta\left(\left\| \mathbf{p}_i - \bar{\mathbf{x}} \right\| \right)$$



Weighted Least Squares

Weighting functions

- Gaussian $\theta(d) = e^{-\frac{d^2}{h^2}}$
  - h is a smoothing parameter
- Wendland function

$$\theta(d) = (1 - d/h)^4 (4d/h + 1)$$

  - Defined in $[0, h]$ and

$$\theta(0) = 1,\ \theta(h) = 0,\ \theta'(h) = 0 \text{ and } \theta''(h)$$

- Singular function

$$\theta(d) = \frac{1}{d^2 + \varepsilon^2}$$

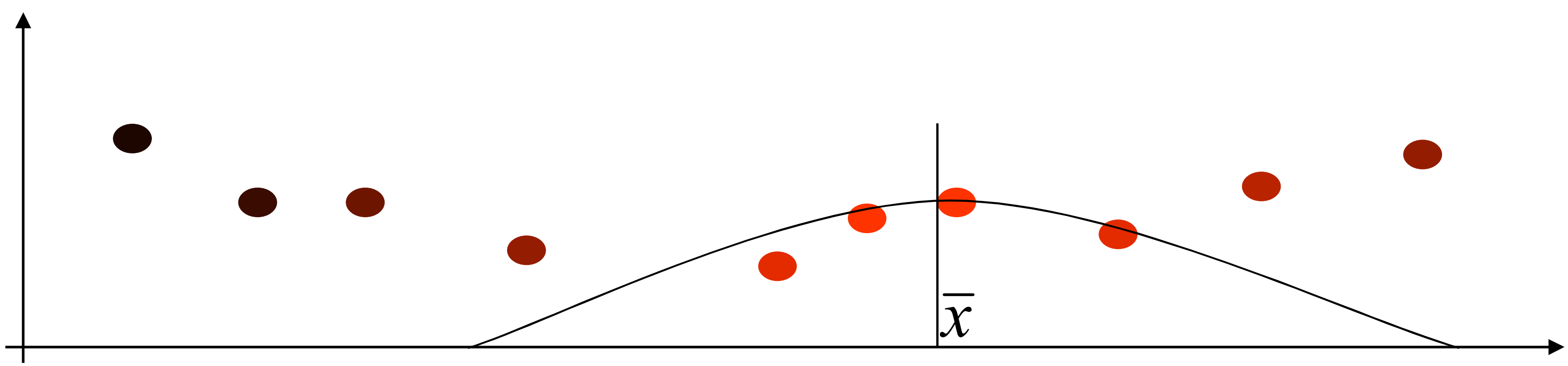
  - For small ε, weights large near $d=0$ (interpolation)

Moving Least Squares

Parametric 1D example

- Principle: "construct" a global function from infinitely many locally weighted functions

$$\mathrm{f}(\mathbf{x}) = \mathrm{f}_{\bar{\mathbf{x}}}(\mathbf{x}), \quad \min_{\mathrm{f}_{\mathbf{x}} \in \Pi_k^d} \sum_{i=0}^{n} \left\| \mathrm{f}(\mathbf{p}_i) - f_i \right\|^2 \theta\left(\left\| \mathbf{p}_i - \bar{\mathbf{x}} \right\| \right)$$



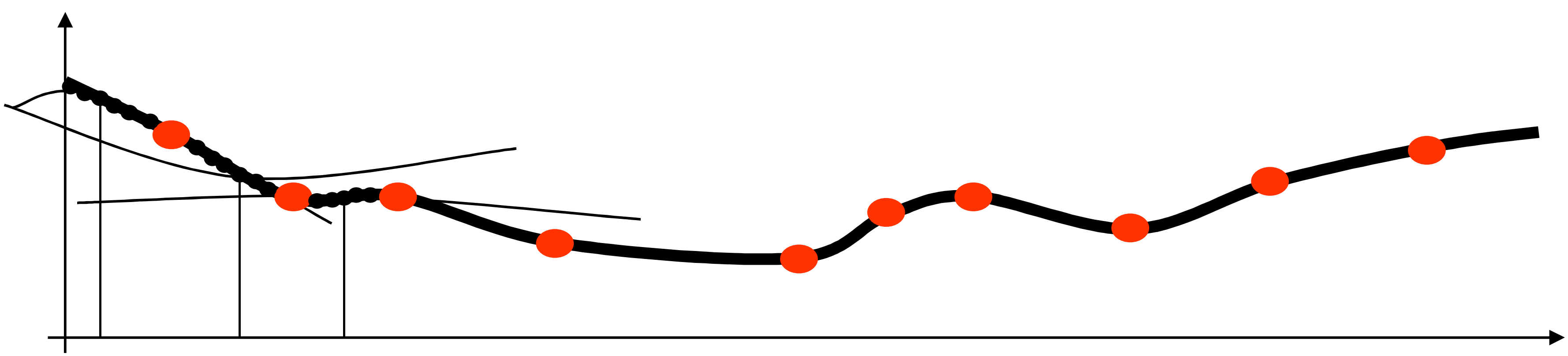
Moving Least Squares

Parametric 1D example

- The infinite set

$$\mathrm{f}(\mathbf{x}) = \mathrm{f}_{\overline{\mathbf{x}}}(\mathbf{x}), \quad \min_{\mathrm{f}_{\mathbf{x}} \in \Pi_k^d} \sum_{i=0}^{n} \left\| \mathrm{f}(\mathbf{p}_i) - f_i \right\|^2 \theta\left(\left\| \mathbf{p}_i - \overline{\mathbf{x}} \right\| \right)$$

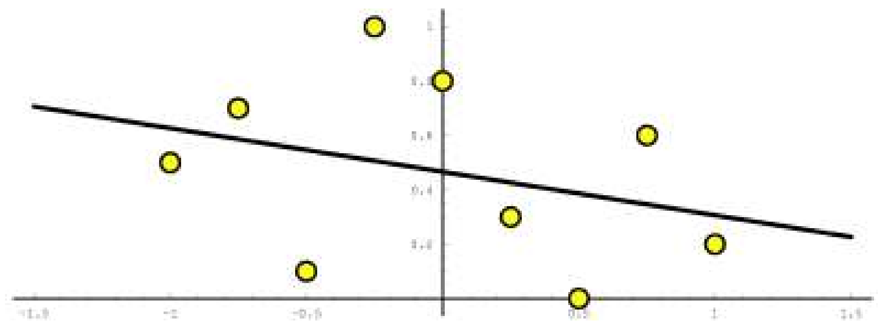
is continuously differentiable if and only if θ is continuously differentiable



LS, MLS and Weight Functions

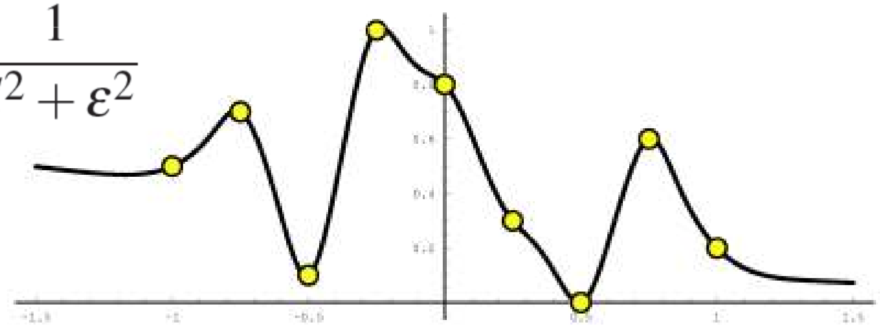
Linear polynomial fit

- Global least squares



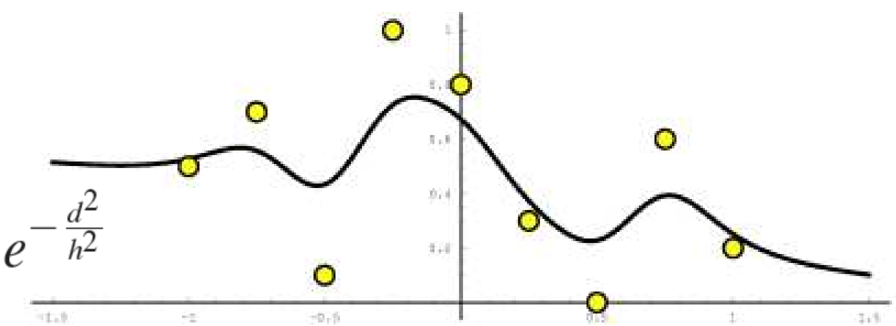
- MLS with (near) singular weight function

$$\theta(d) = \frac{1}{d^2 + \varepsilon^2}$$



- MLS with approximating weight function

$$\theta(d) = e^{-\frac{d^2}{h^2}}$$

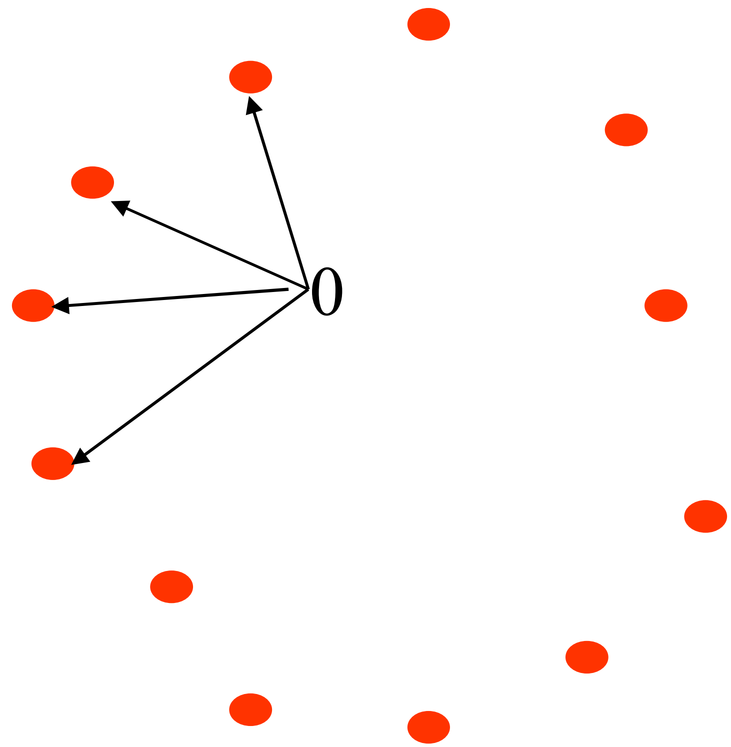


Implicit Surface Reconstruction

Distance Field Reconstruction

2D example

- Idea: construct a distance field on the points
- Implicit function
  $$\mathrm{f}(\mathbf{p}_i) = 0$$
  for the points $\mathbf{p}_i$
- Trivial solution $f = 0$
- Requires additional constraints



Distance Field Reconstruction

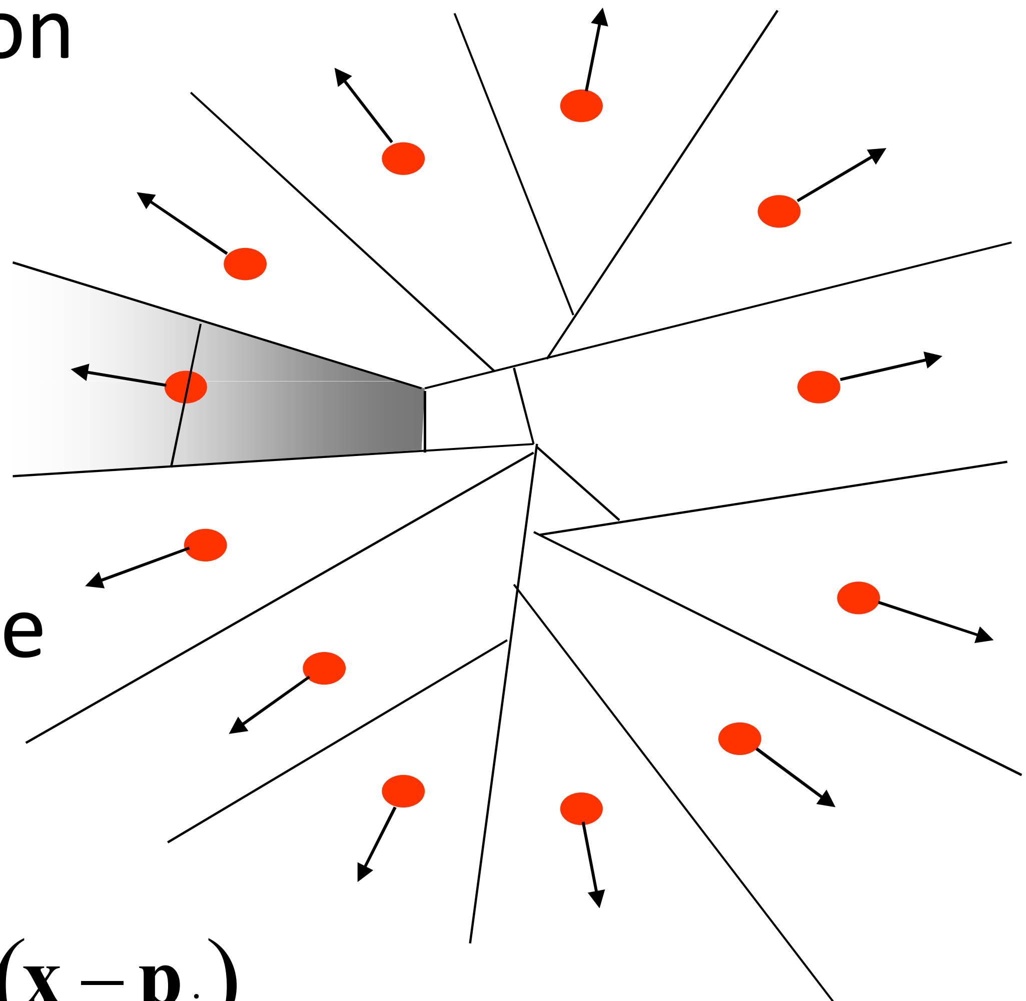
[Hoppe et al. 1992]

- Linear distance function per point
  - Direction is defined by surface normal

    $$f_i(\mathbf{x}) = \mathbf{n}_i \cdot (\mathbf{x} - \mathbf{p}_i)$$

- Distance in space is the minimum of all local distance functions

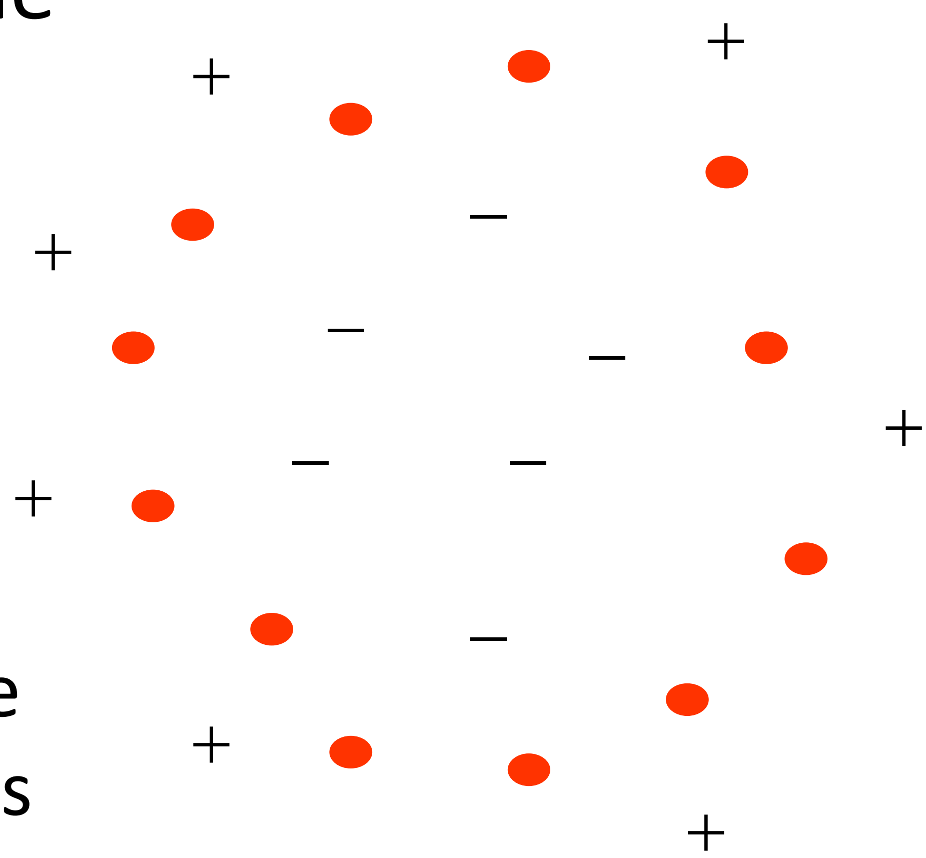
$$f(\mathbf{x}) = \min_i f_i(\mathbf{x}) = \min_i \mathbf{n}_i \cdot (\mathbf{x} - \mathbf{p}_i)$$



Distance Field Reconstruction

Inside + outside point constraints

- Additional data to define inside and outside
- Basic idea [Turk and O'Brien 1999]
  - Insert additional value constraints manually
  - These constraints can be added as soft constraints with low(er) weight



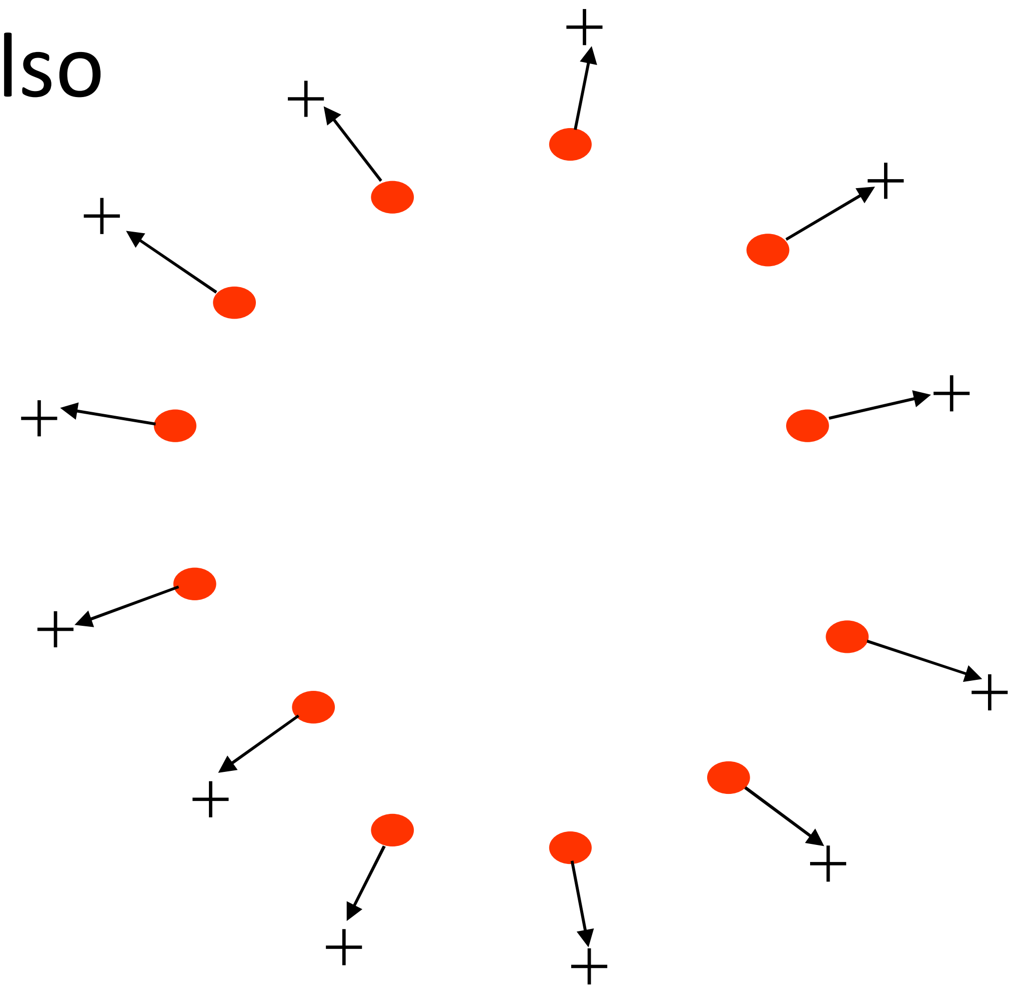
Distance Field Reconstruction

Inside + outside point constraints

- This information can also be obtained from surface normals

$$f(\mathbf{p}_i + \alpha \mathbf{n}_i) = \alpha$$

- Some acquisition devices provide normals
- If not, they must be locally approximated



Distance Field Reconstruction

Inside + outside point constraints

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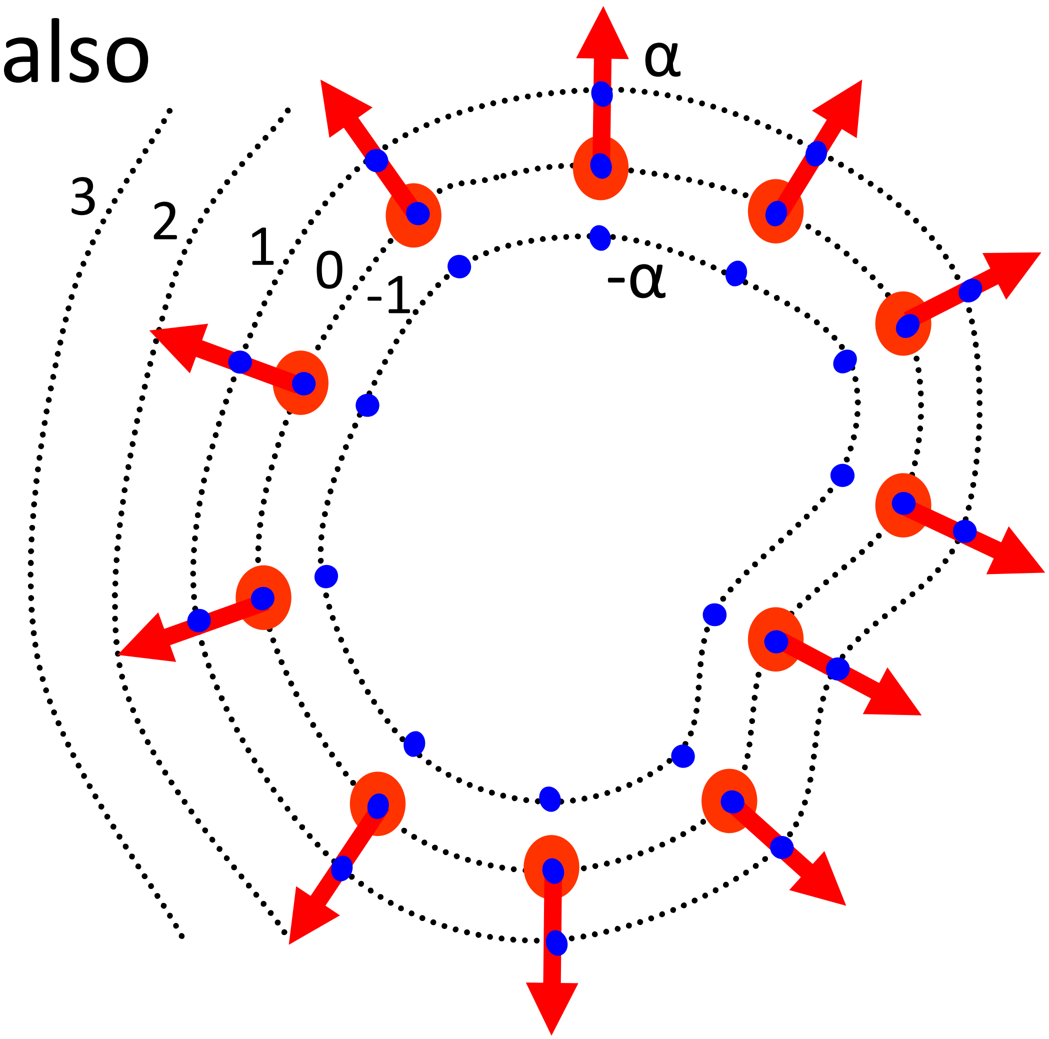
- Some acquisition devices provide normals
- If not, they must be locally approximated



Distance Field Reconstruction

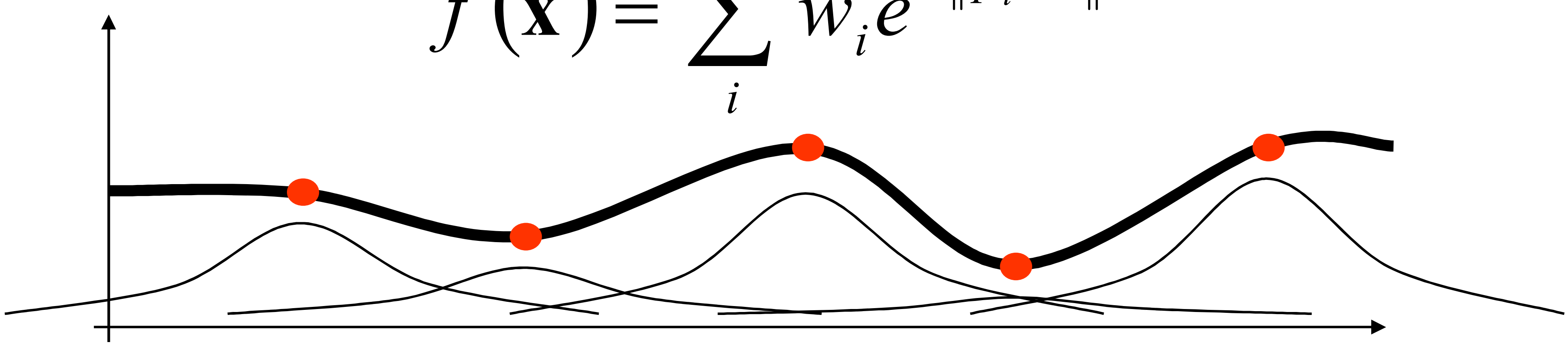
Radial basis functions (RBFs)

- Similar to parametric case
- Given points and normals $\mathbf{p}_i, \mathbf{n}_i$ construct a function with

$$\mathrm{f}(\mathbf{p}_i) = 0, \quad \mathrm{f}(\mathbf{p}_i + \alpha \mathbf{n}_i) = \alpha$$

- Possible solution: Gaussian RBFs

$$f(\mathbf{x}) = \sum_i w_i e^{-\|p_i - \mathbf{x}\|^2}$$



Distance Field Reconstruction

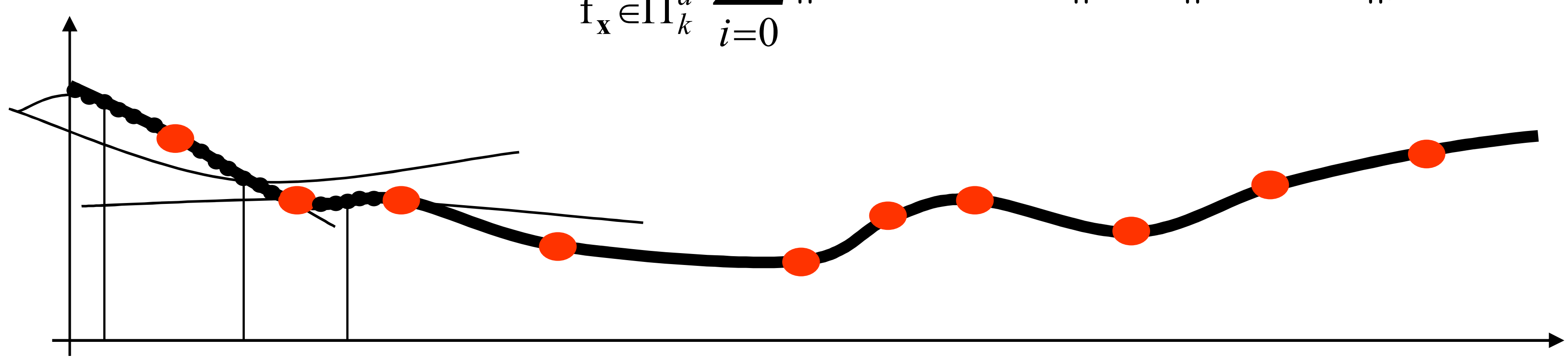
Moving least squares (MLS)

- Given points and normals $\mathbf{p}_i, \mathbf{n}_i$ construct a function with

$$\mathrm{f}\left(\mathbf{p}_i\right) = 0, \quad \mathrm{f}\left(\mathbf{p}_i + \alpha\mathbf{n}_i\right) = \alpha$$

using the moving least squares technique

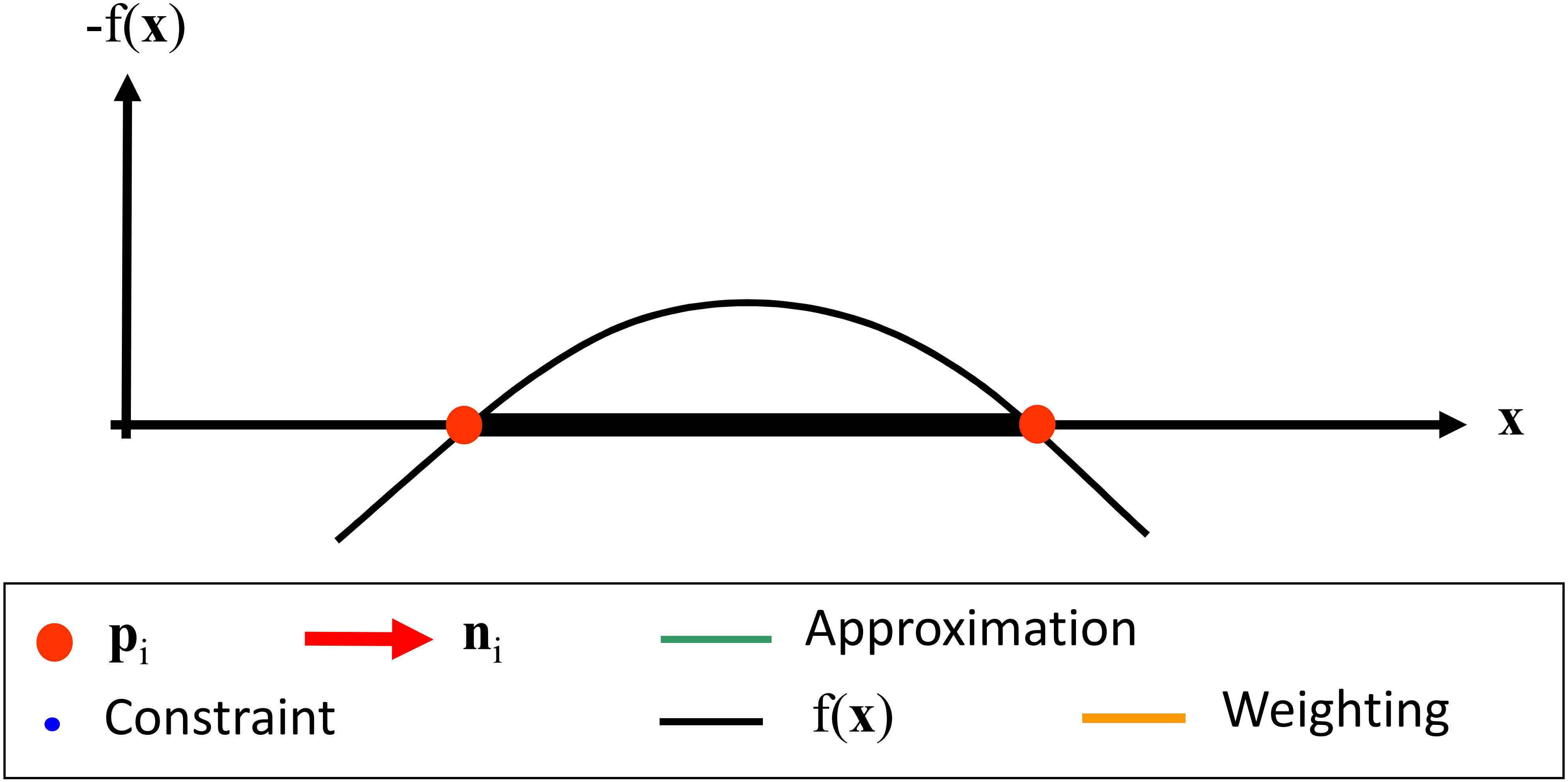
$$\mathrm{f}\left(\mathbf{x}\right) = \mathrm{f}_{\overline{\mathbf{x}}}\left(\mathbf{x}\right), \quad \min_{\mathrm{f}_{\mathbf{x}} \in \Pi_k^d} \sum_{i=0}^{n} \left\| \mathrm{f}(\mathbf{p}_i) - f_i \right\|^2 \theta\left(\left\|\mathbf{p}_i - \overline{\mathbf{x}}\right\|\right)$$



MLS Distance Field

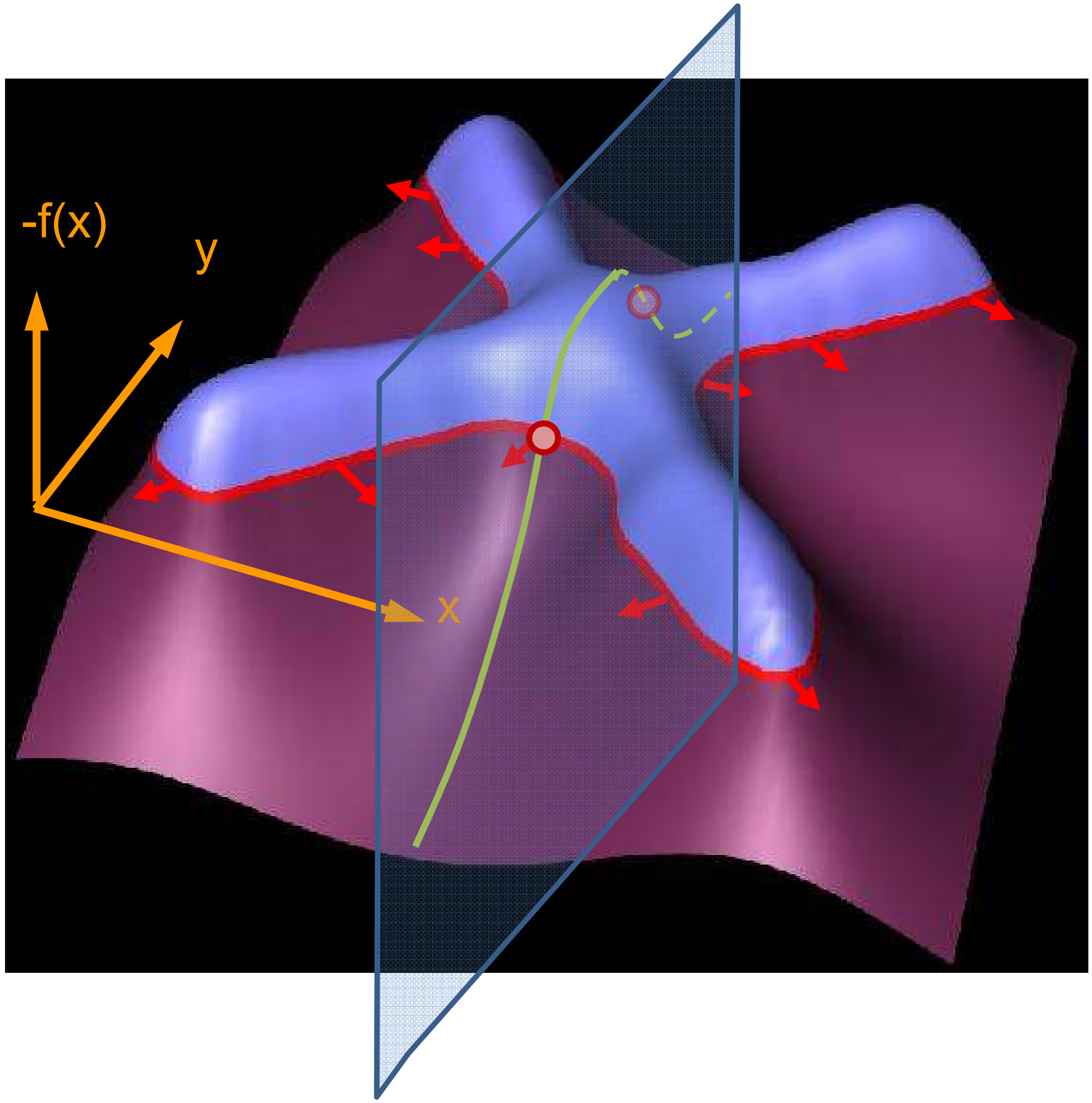
1D example

- One dimensional Implicit function



MLS Distance Field

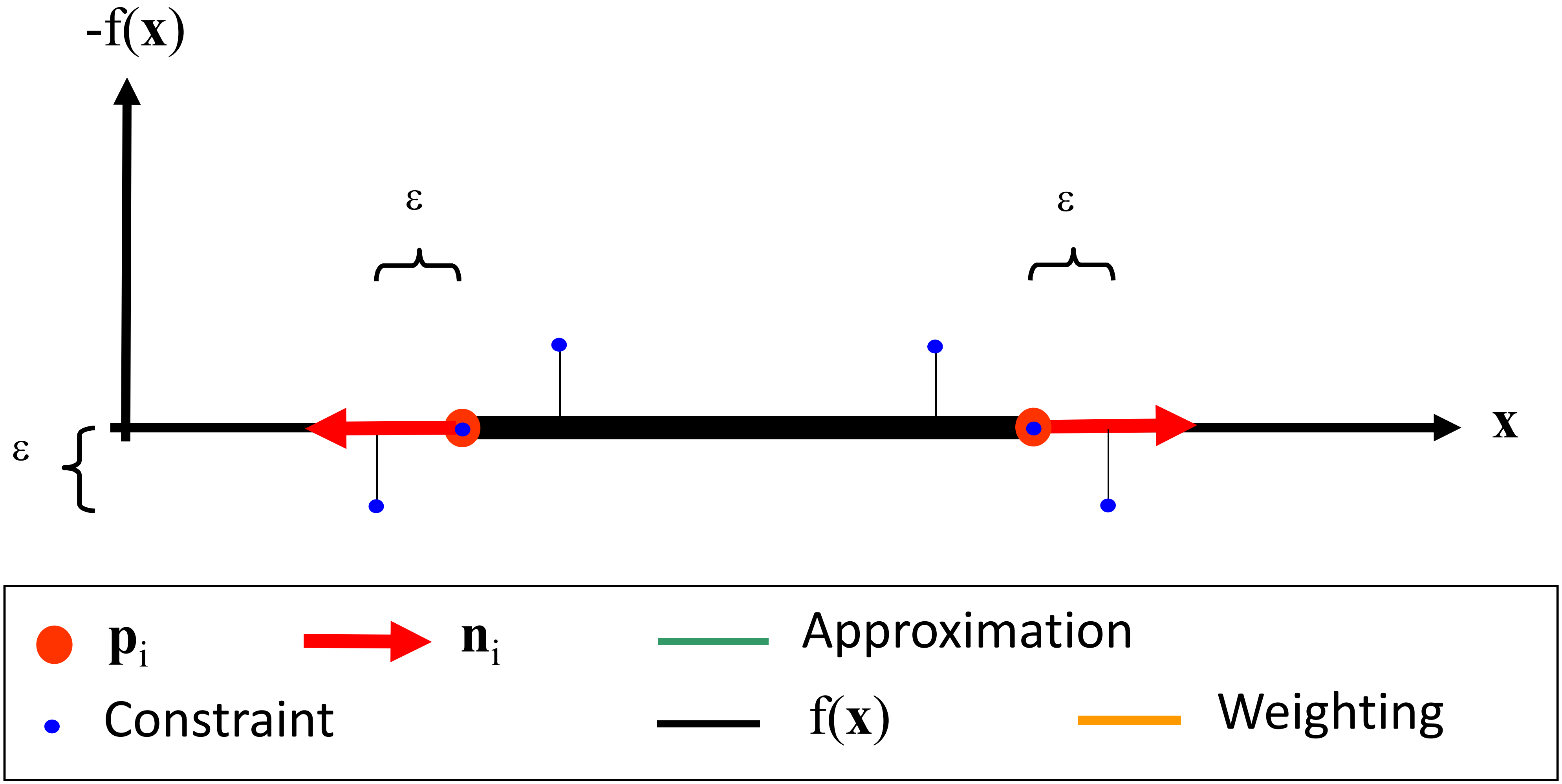
1D slice of a 2D height field



MLS Distance Field

1D example

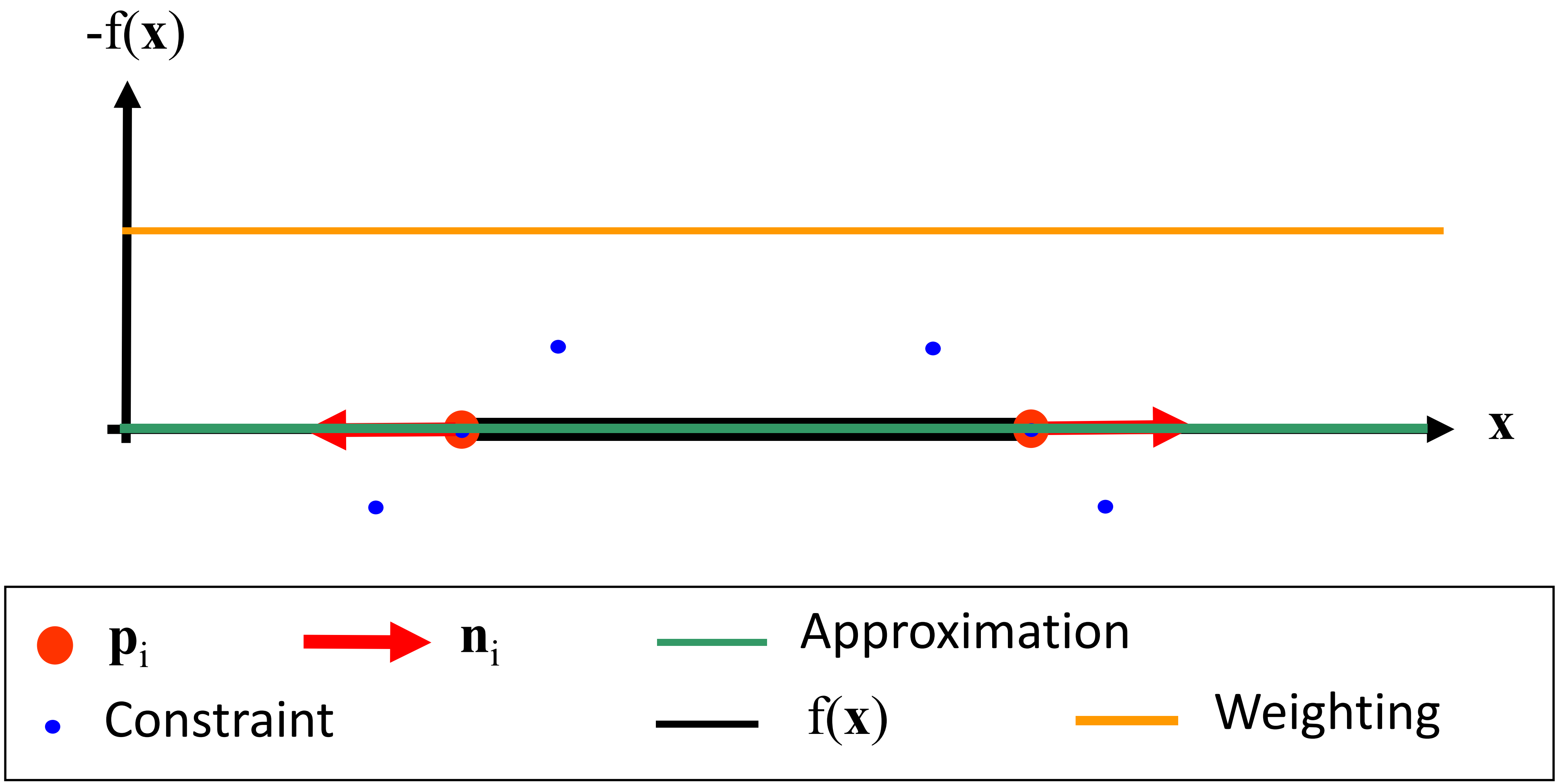
- Adding inside + outside constraints



MLS Distance Field

1D example

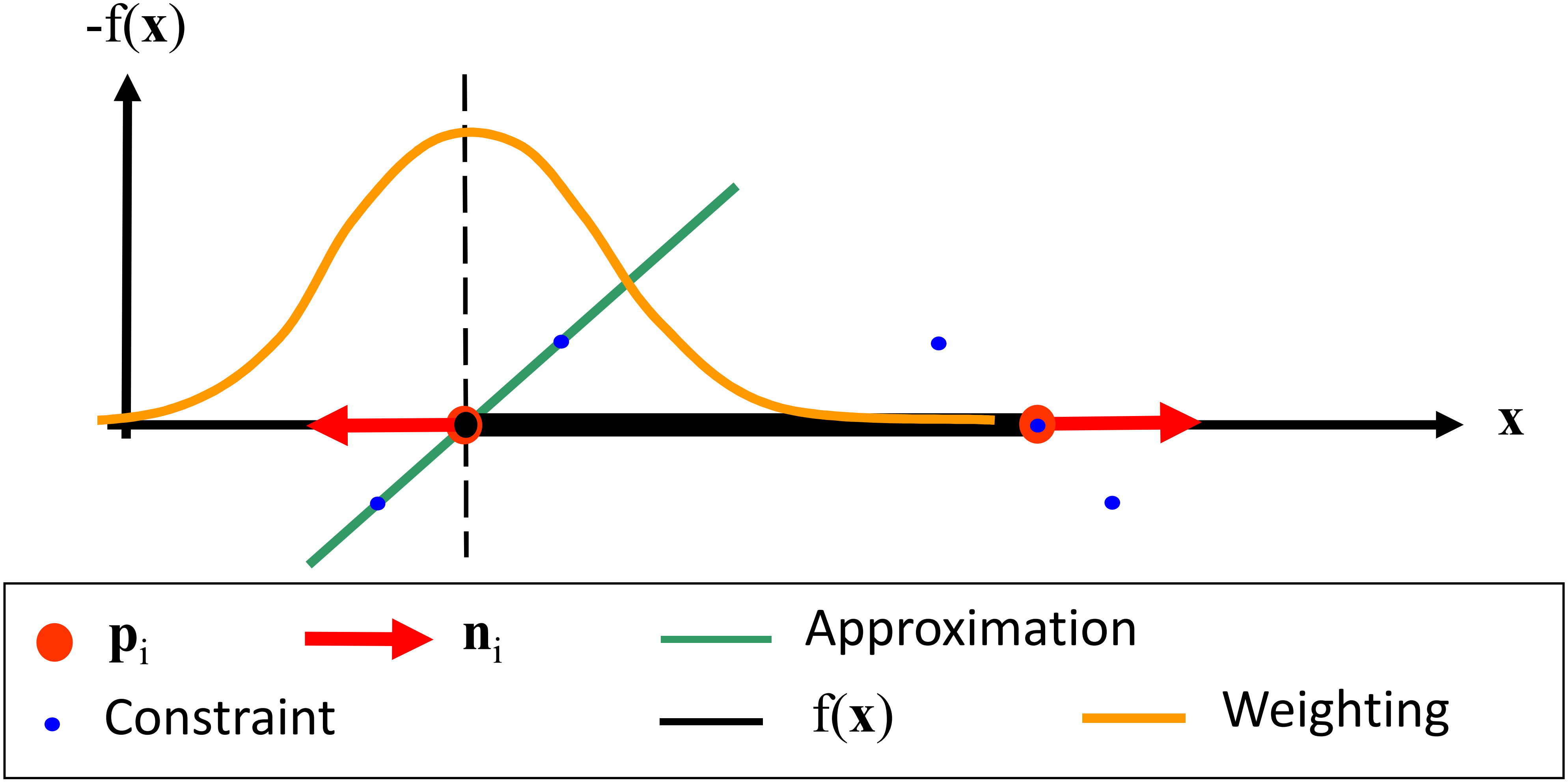
- Linear polynomial fit (uniform weights)



MLS Distance Field

1D example

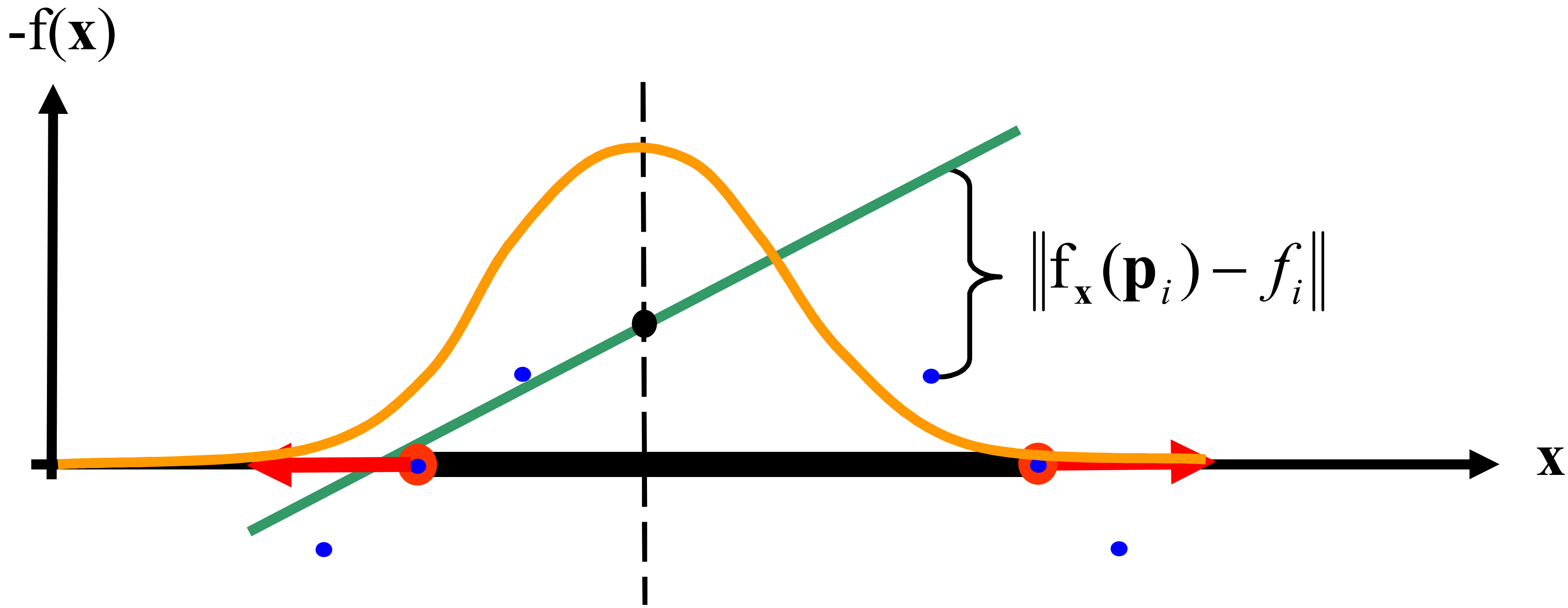
- Linear polynomial fit (Gaussian weights)

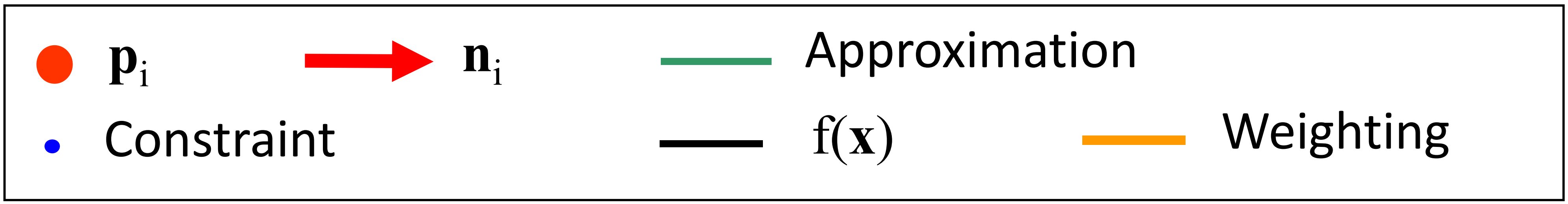


MLS Distance Field

1D example

- Linear polynomial fit (Gaussian weights)

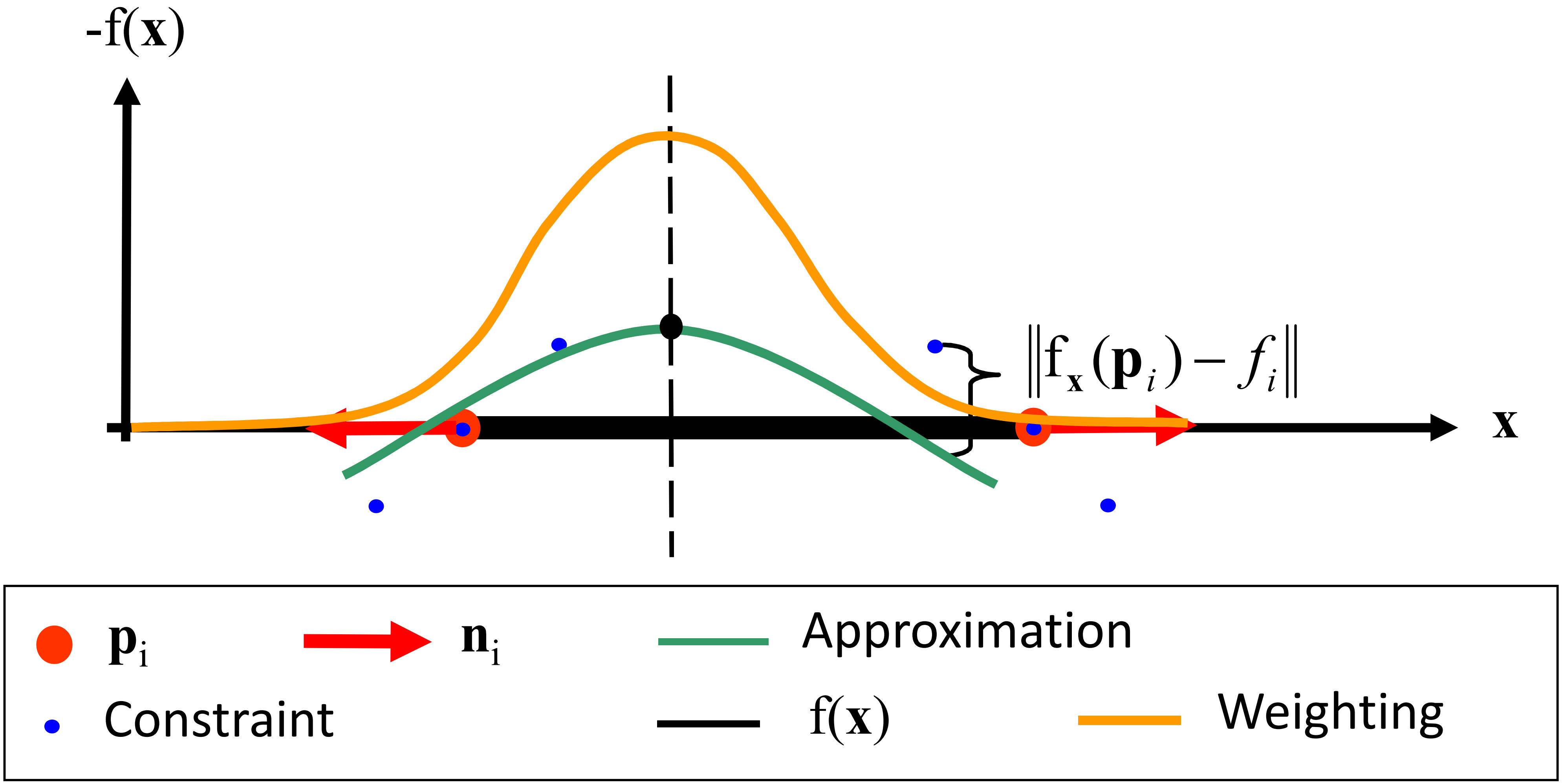




MLS Distance Field

1D example

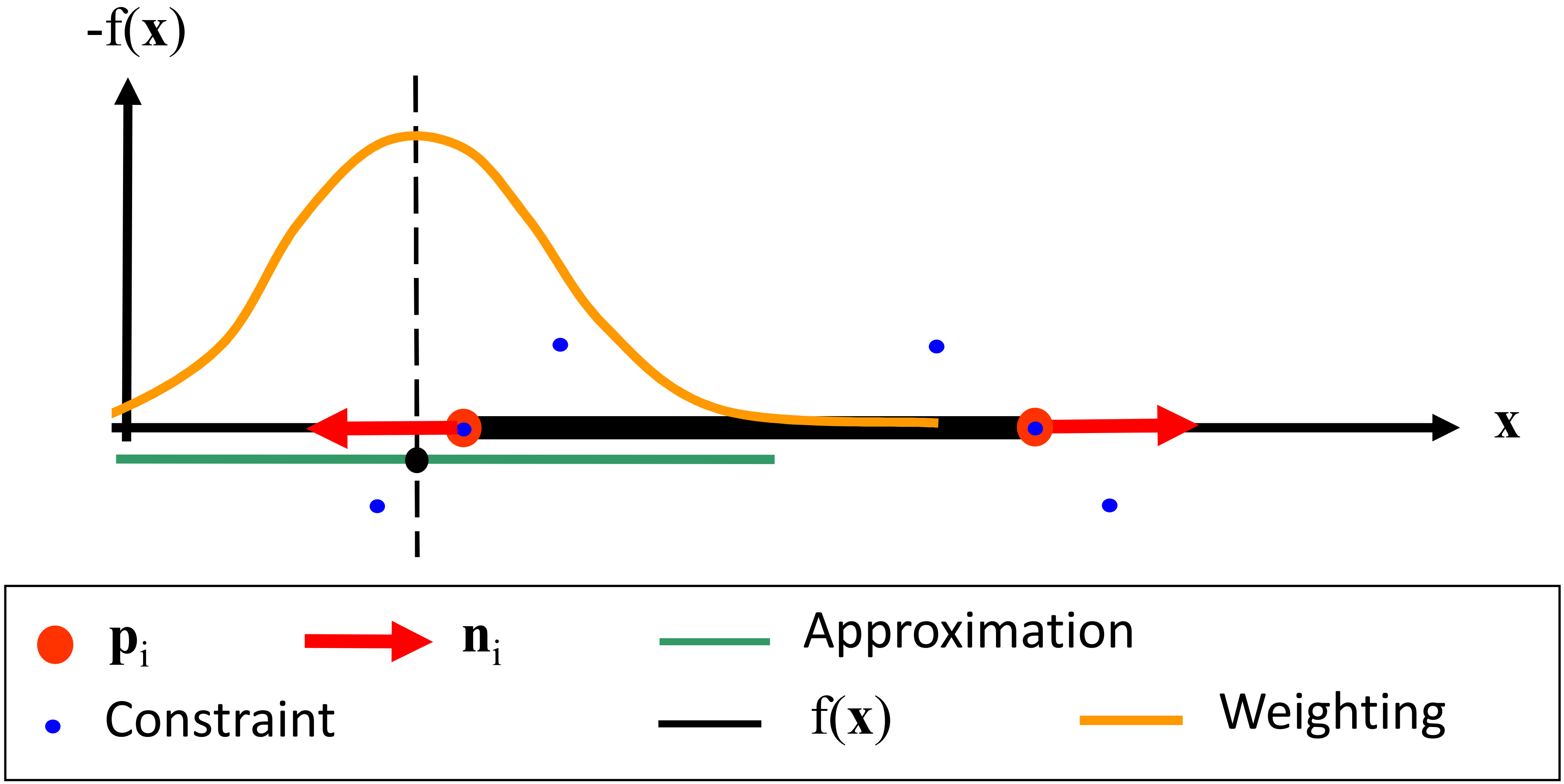
- Quadratic polynomial fit (Gaussian weights)



MLS Distance Field

1D example

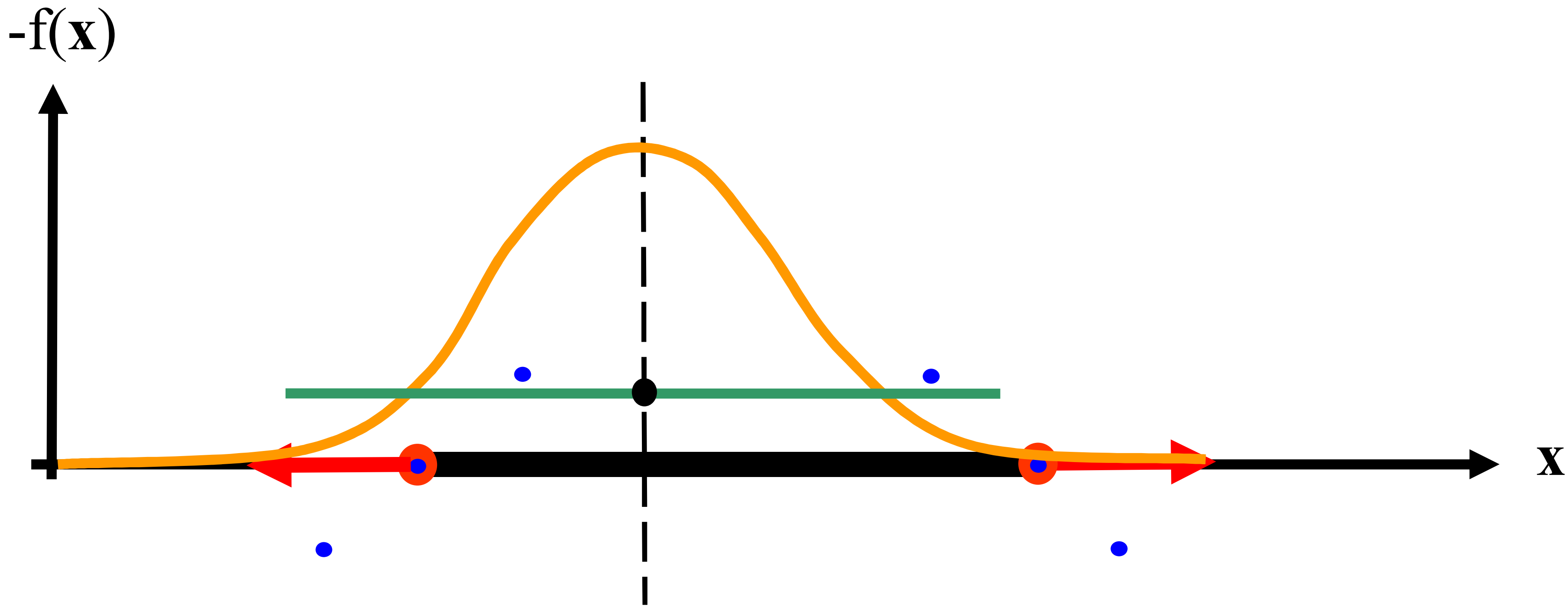
- Constant polynomial fit (Gaussian weights)

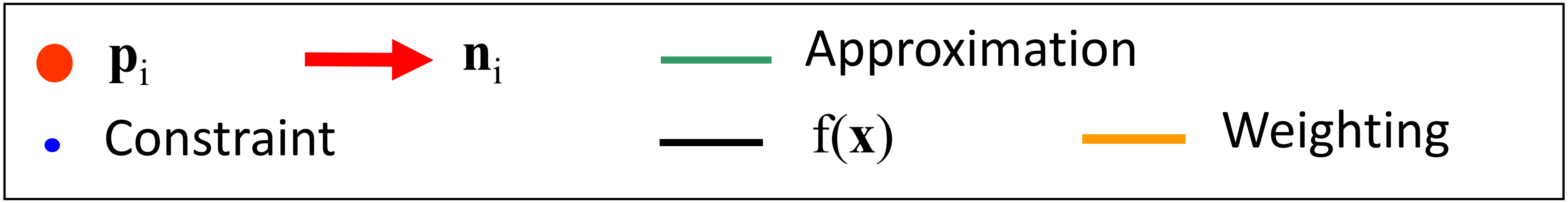


MLS Distance Field

1D example

- Constant polynomial fit (Gaussian weights)

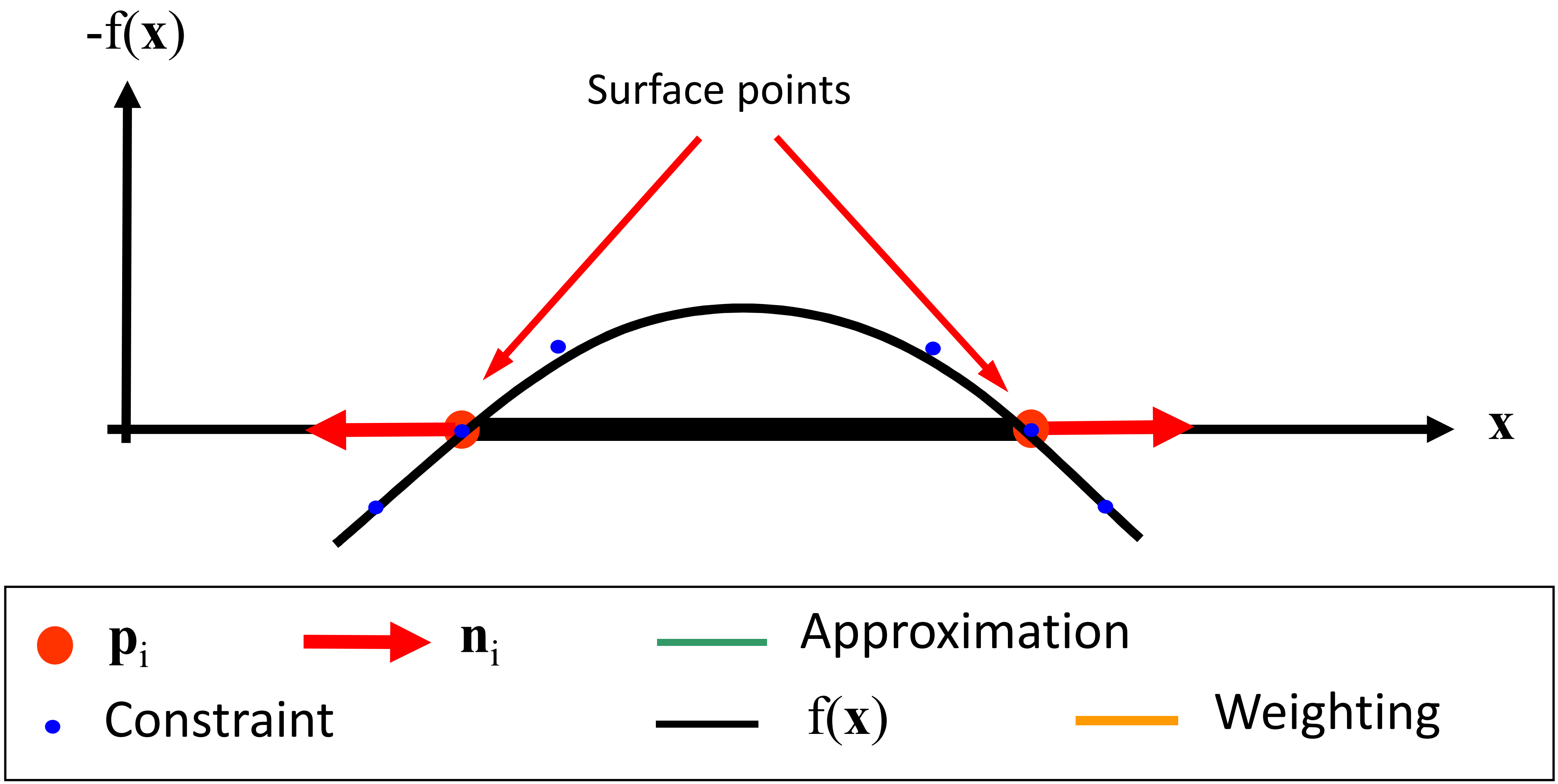




MLS Distance Field

1D example

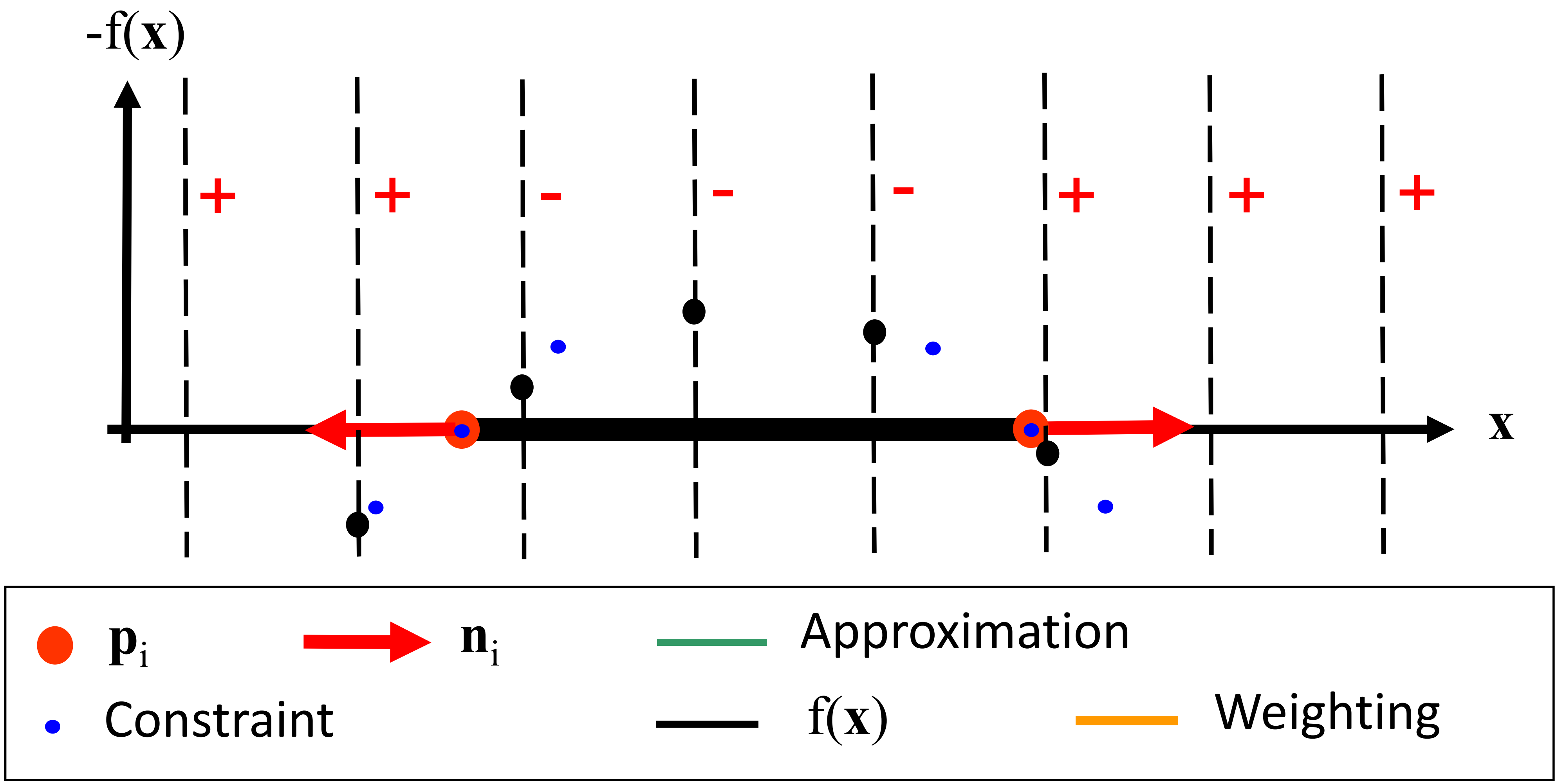
- MLS approximation results



MLS Distance Field

1D example

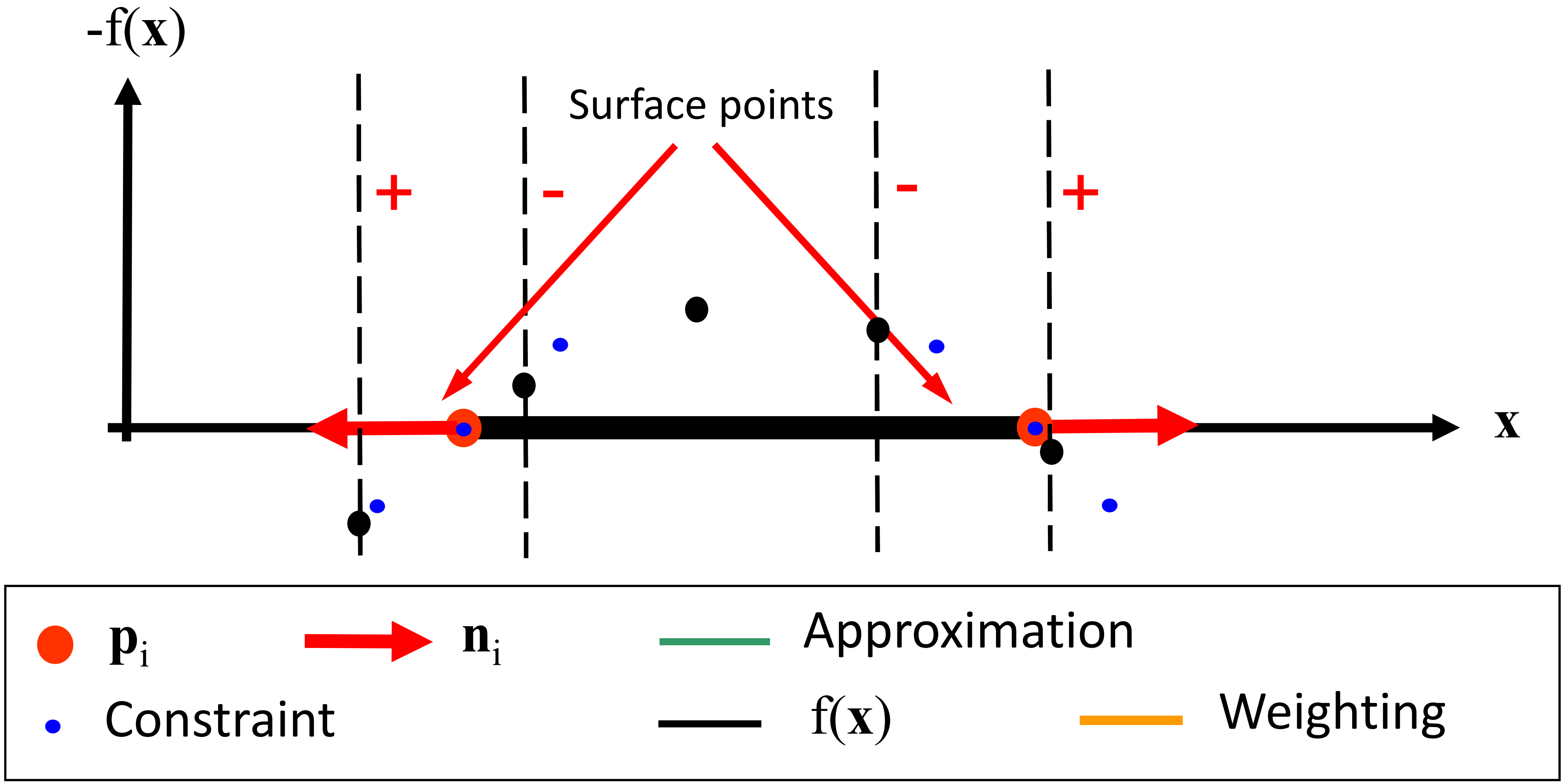
- Discrete evaluation with marching cubes (3D)



MLS Distance Field

1D example

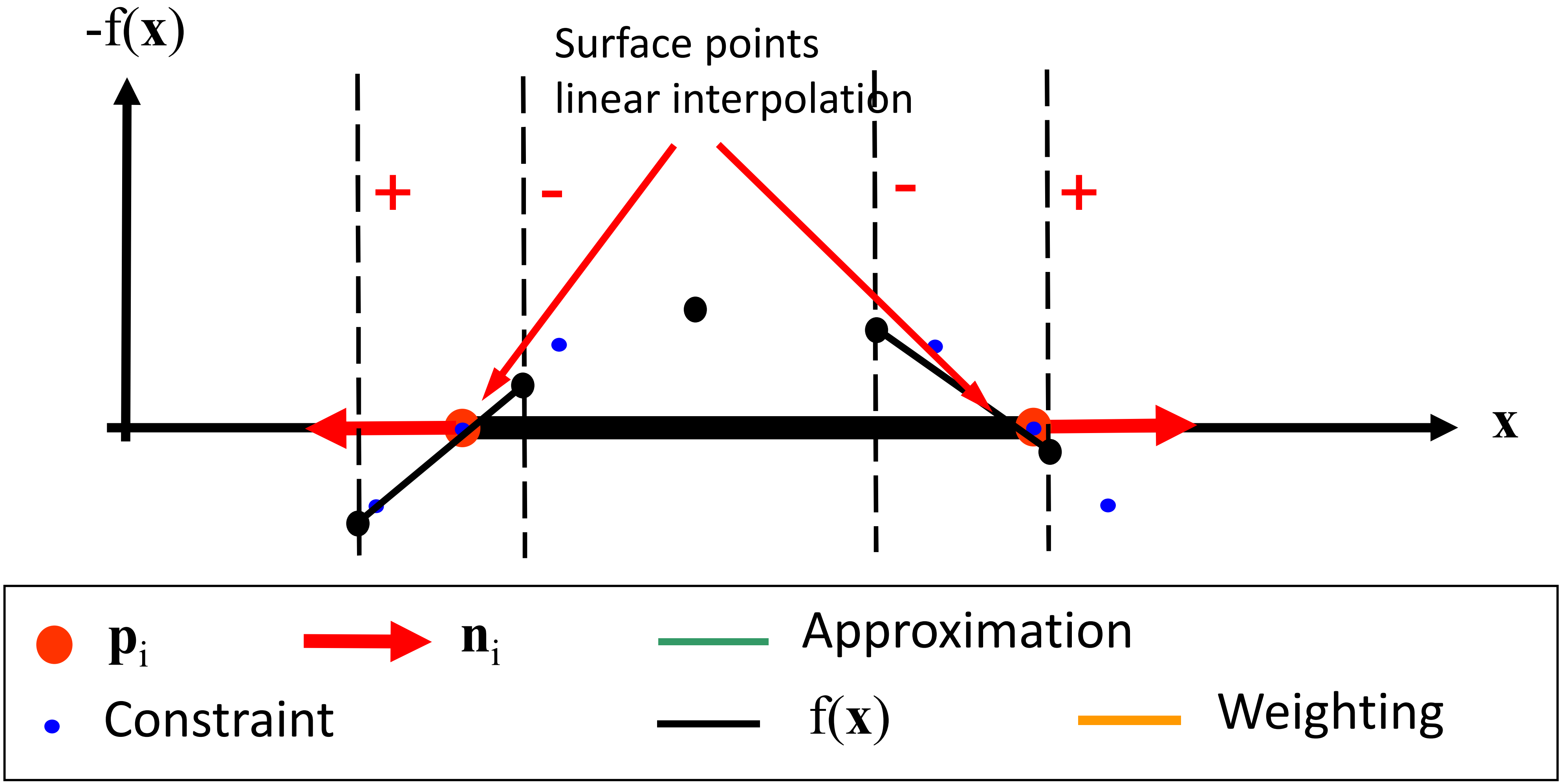
- Discrete evaluation with marching cubes (3D)



MLS Distance Field

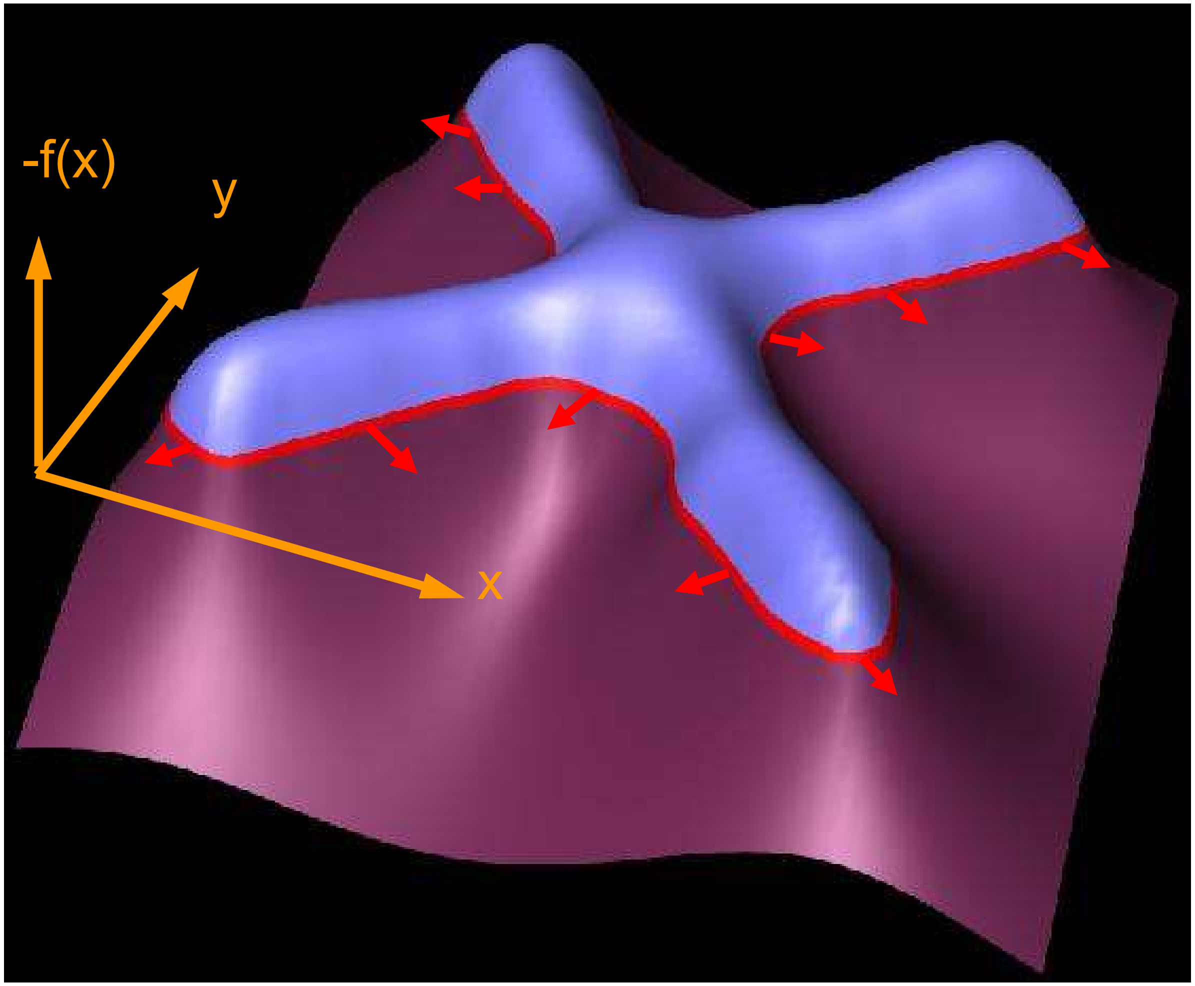
1D example

- Discrete evaluation with marching cubes (3D)



MLS Distance Field

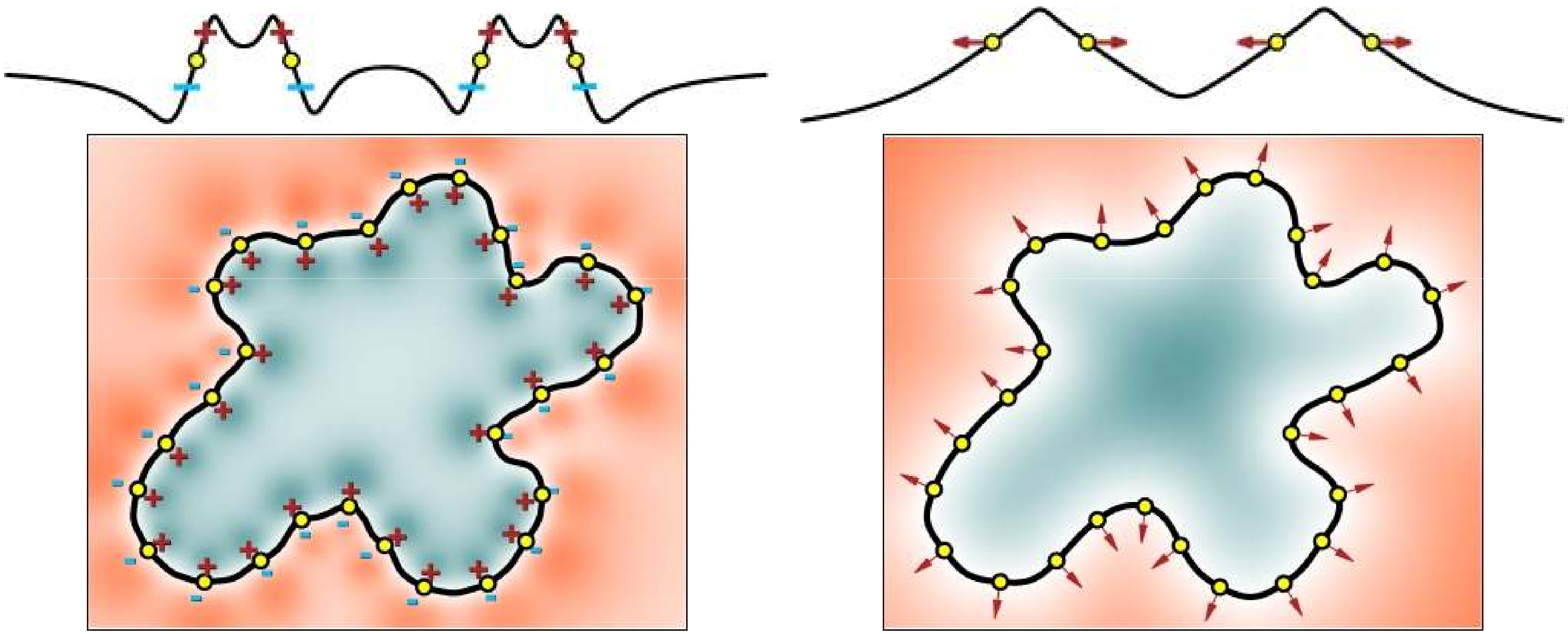
2D Illustration



MLS Distance Field

Extensions

- Point constraints vs. true normal constraints

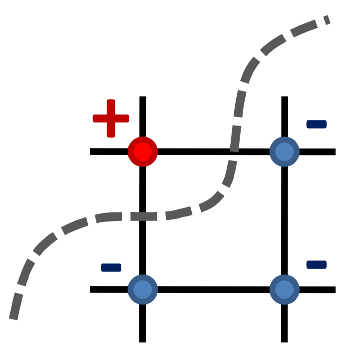


- Details: Shen, C., O'Brien, J. F., Shewchuk J. R., "**Interpolating and Approximating Implicit Surfaces from Polygon Soup.**" *Proceedings of ACM SIGGRAPH 2004*, Los Angeles, California, August 8-12.

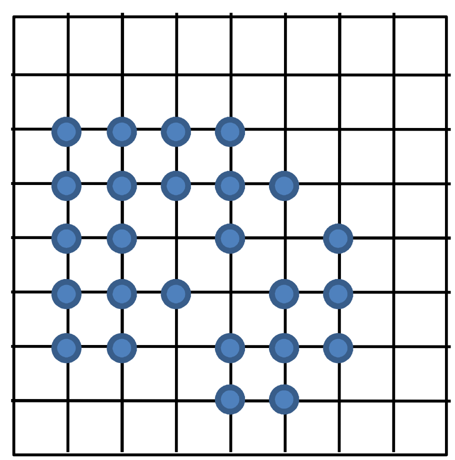
Tessellation of implicit surfaces

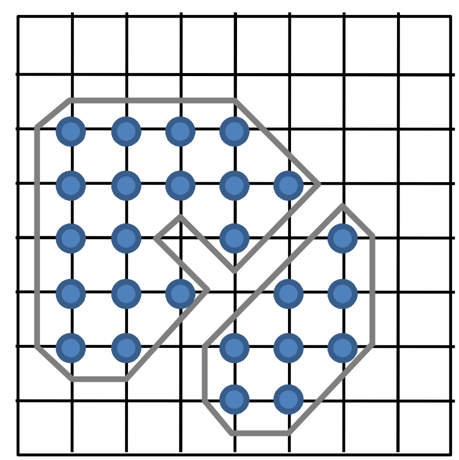
Tessellation

- Want to approximate an implicit surface with a mesh
  - For rendering, further processing
- Can‘t explicitly compute all the roots
  - Infinite amount (the whole surface)
  - The expression of the implicit function may be complicated
- Solution: find approximate roots by trapping the implicit surface in a grid (lattice)



● $f(\mathbf{p})<0$

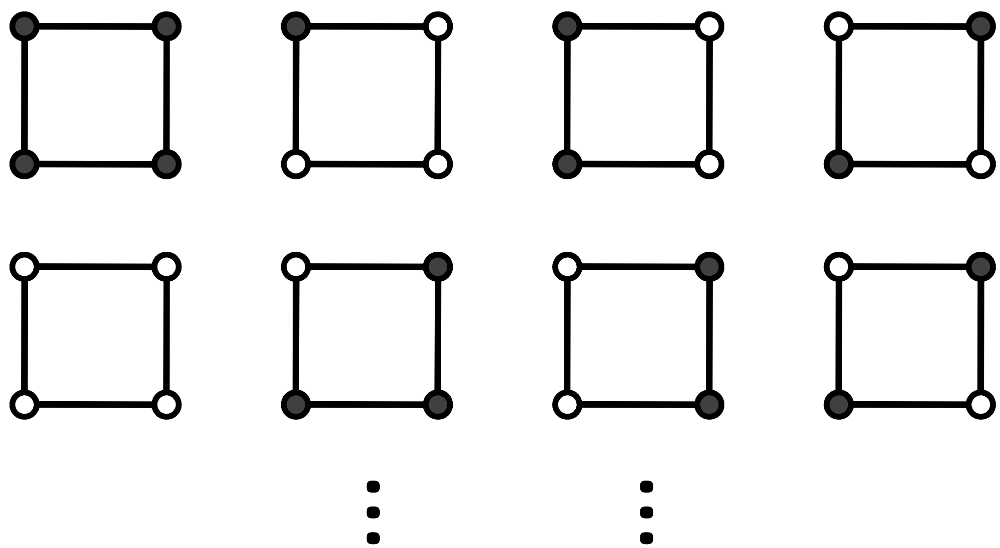




Tessellation

2D grid

- 16 different configurations in 2D
- 4 equivalence classes (up to rotational and reflection symmetry + complement)



Tessellation

2D grid

- 16 different configurations in 2D
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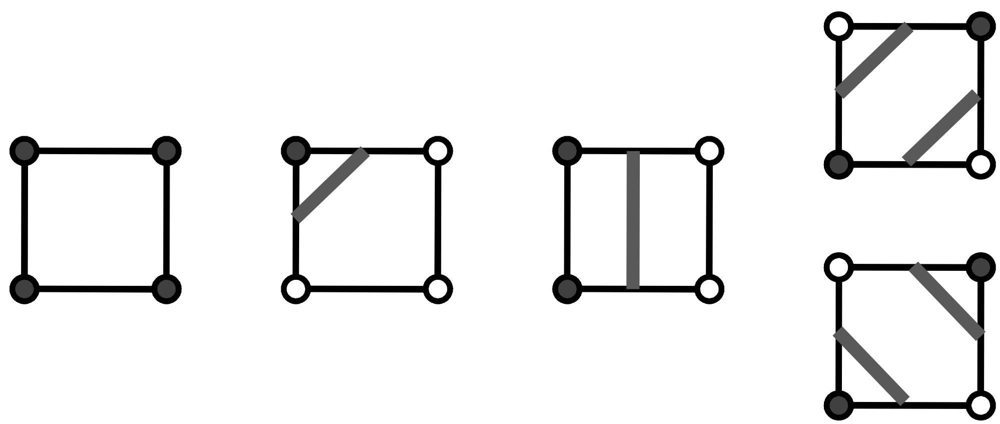
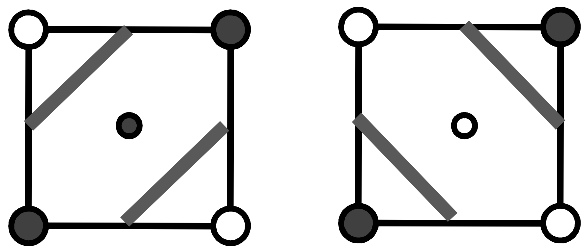


case 1 case 2 case 3 case 4

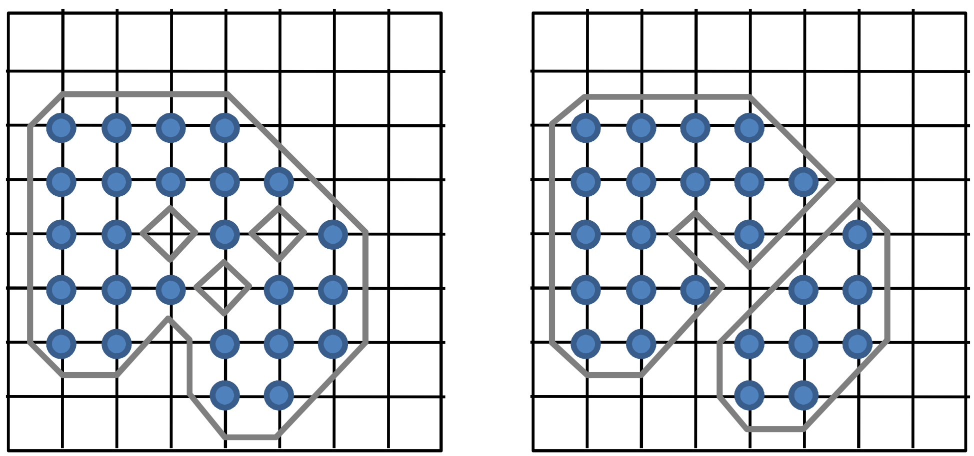
Tessellation

2D grid, consistency

- Case 4 is ambiguious:



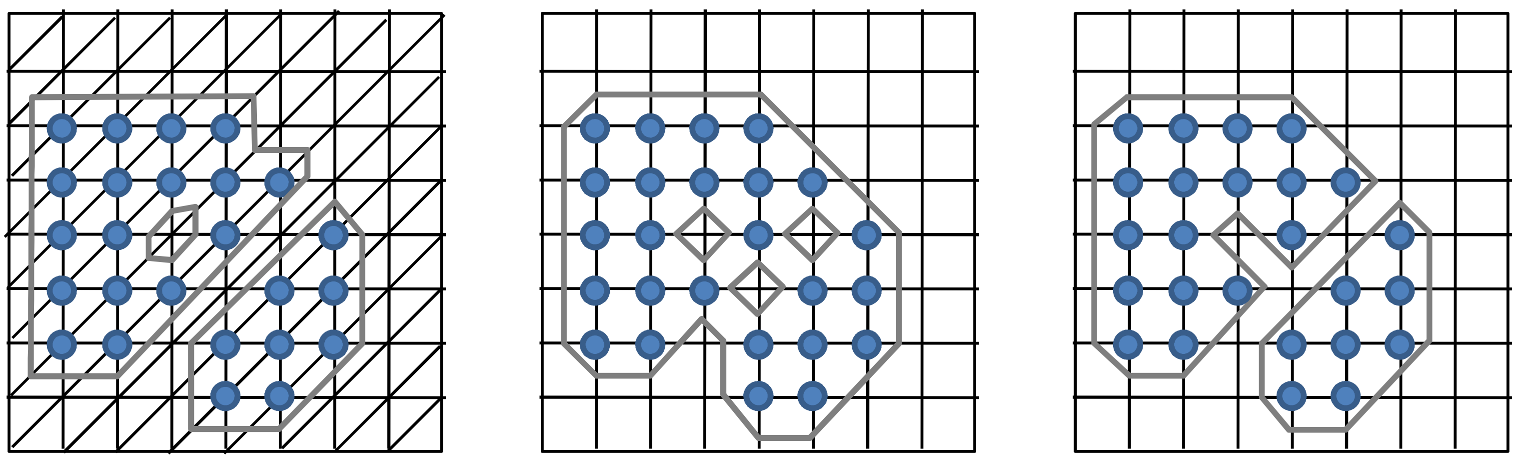
- Always pick consistently to avoid problems with the resulting mesh



Tessellation

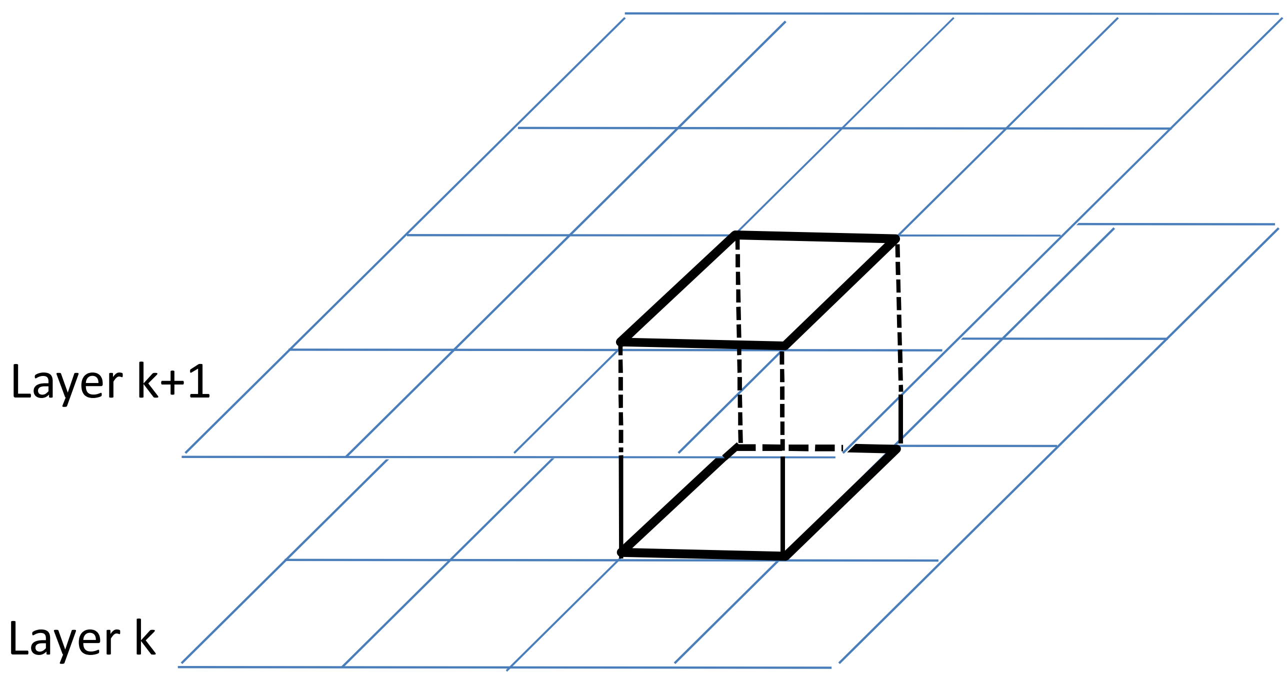
2D triangle grid

- No ambiguity if we have triangles instead of squares
- However, it is still unknown what the true surface is!



Tessellation

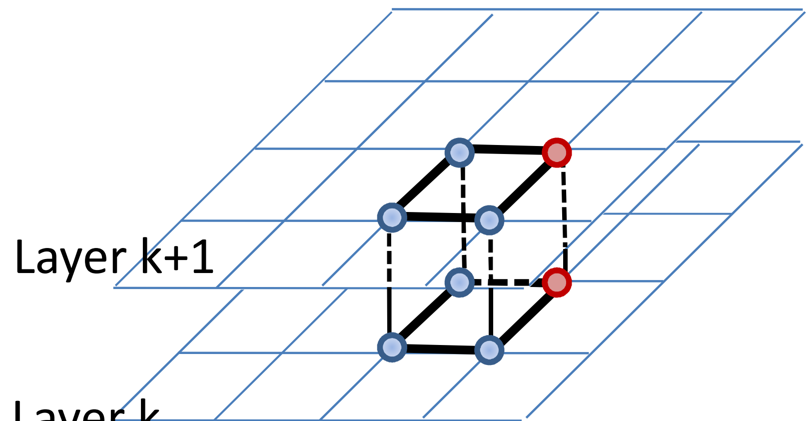
3D – Marching Cubes



Tessellation

3D – Marching Cubes

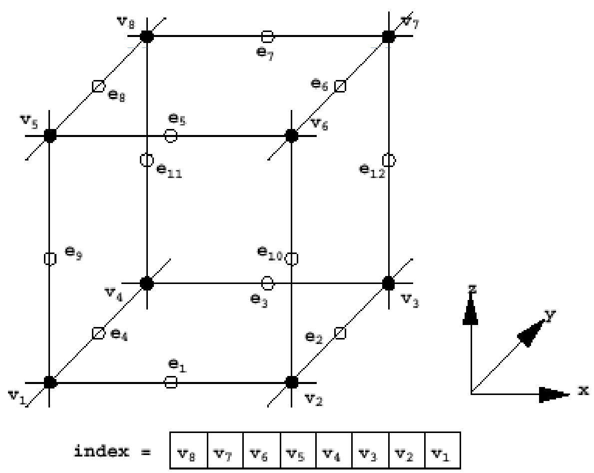
- Marching Cubes (Lorensen and Cline 1987)
  1. Load 4 layers of the grid into memory
  2. Create a cube whose vertices lie on the two middle layers
  3. Classify the vertices of the cube according to the implicit function (inside, outside or on the surface)

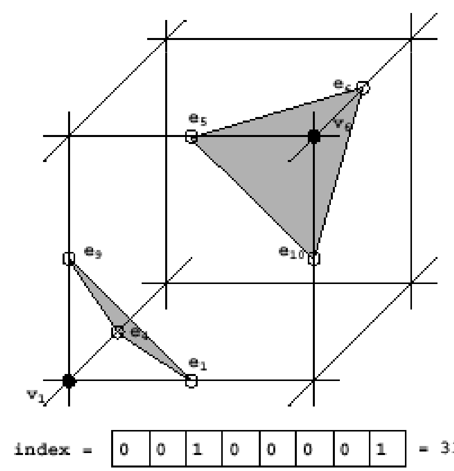


Tessellation

3D – Marching Cubes

4. Compute case index. We have 2^8= 256 cases (0/1 for each of the eight vertices) – can store as 8 bit (1 byte) index.

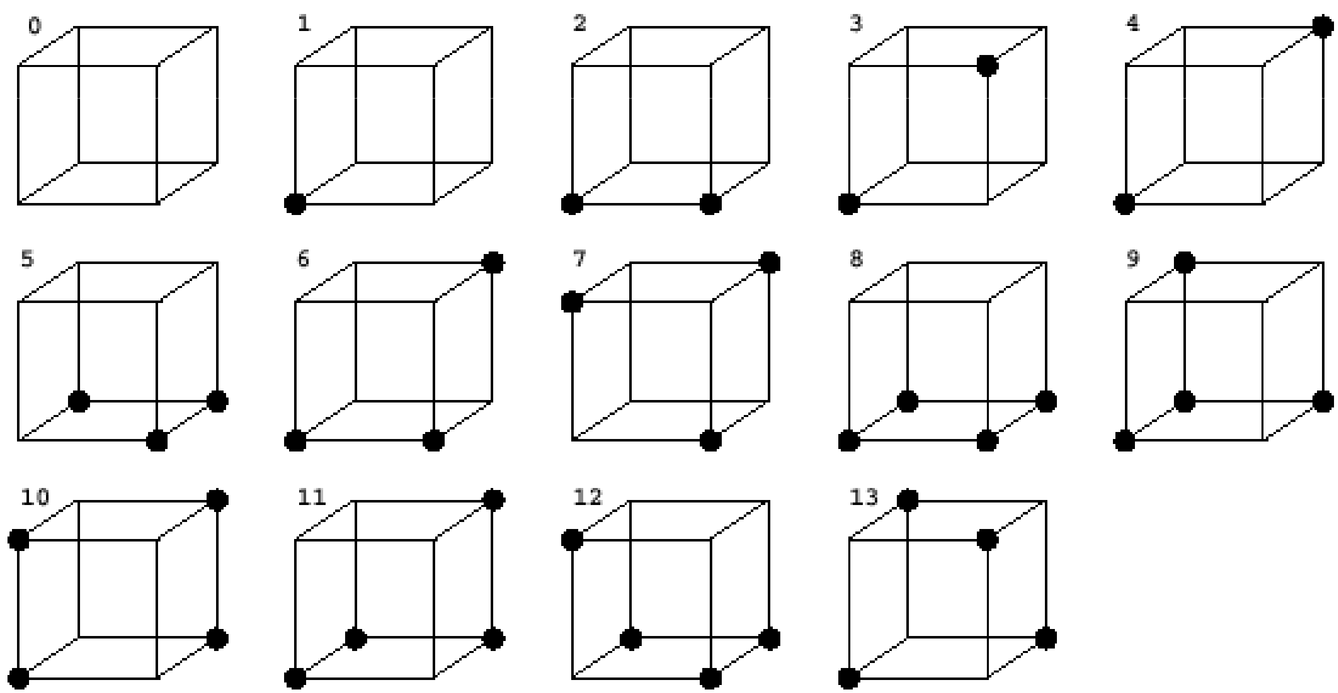




Tessellation

3D – configurations

- We have 14 equivalence classes (by rotation, reflection and complement)

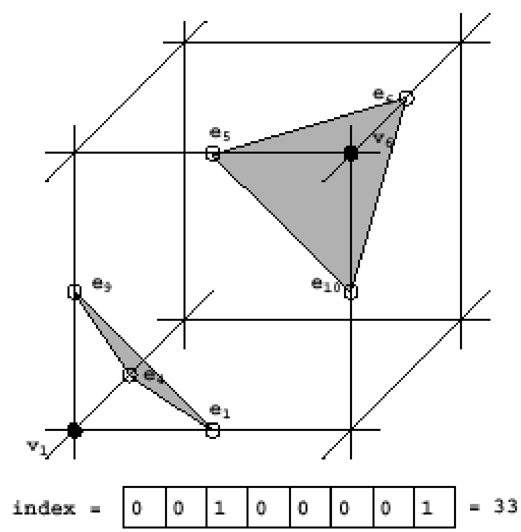


Tessellation

3D – Marching Cubes

5. Using the case index, retrieve the connectivity in the look-up table

- Example: the entry for index 33 in the look-up table indicates that the cut edges are e_1; e_4; e_5; e_6; e_9 and e_{10} ; the output triangles are (e_1; e_9; e_4) and (e_5; e_{10}; e_6).



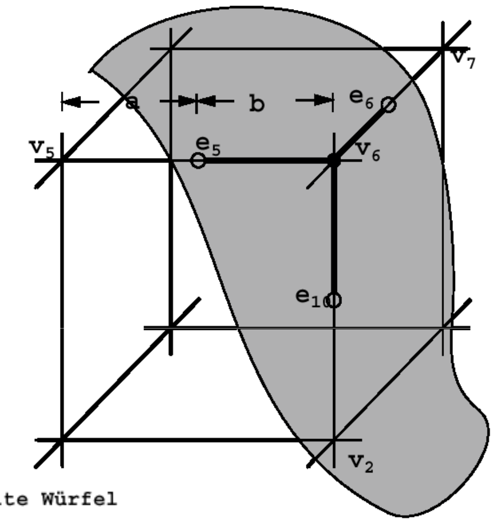
Tessellation
3D – Marching Cubes

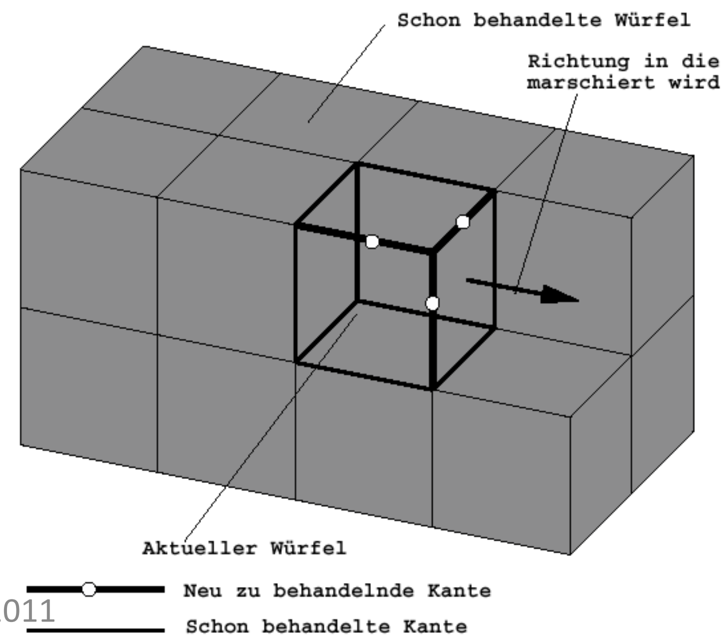
6. Compute the position of the cut vertices by linear interpolation:

$$\mathbf{v}_s = \alpha \mathbf{v}_a + (1-\alpha)\mathbf{v}_b$$

$$\alpha = \frac{\mathrm{f}(\mathbf{v}_b)}{\mathrm{f}(\mathbf{v}_b) - \mathrm{f}(\mathbf{v}_a)}$$

7. Compute the vertex normals
8. Move to the next cube

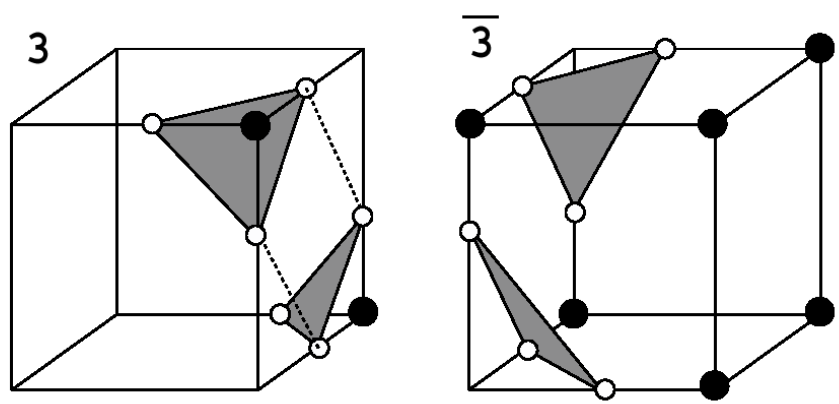




Tessellation

3D – configurations, consistency

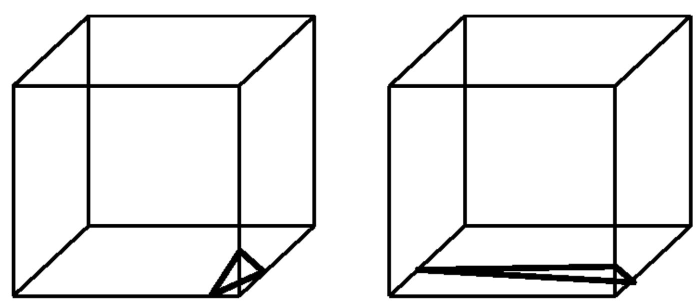
- Have to make consistent choices for neighboring cubes
- Prevent " holes" in the triangulation



Tessellation

Grid-Snapping

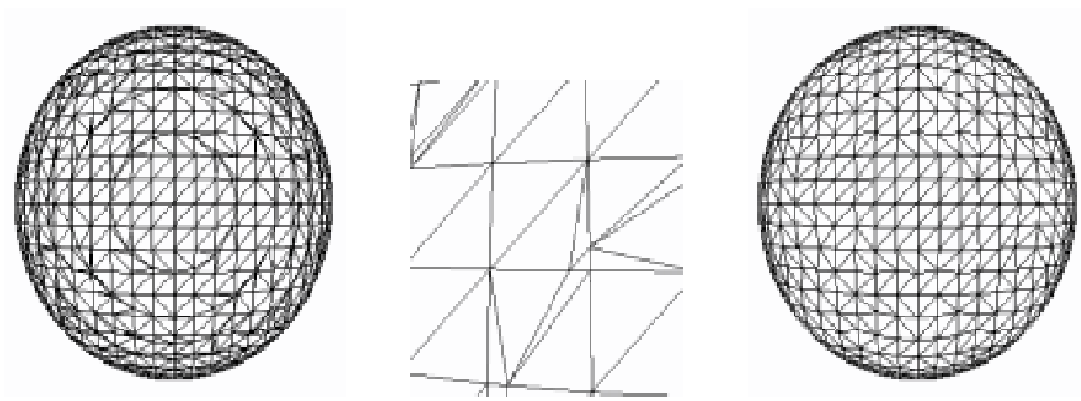
- Problems with short triangle edges
  - When the surface intersects the cube close to a corner, the resulting tiny triangle doesn‘t contribute much area to the mesh
  - When the intersection is close to an edge of the cube, we get skinny triangles (bad aspect ratio)
- Triangles with short edges waste resources but don‘t contribute to the surface mesh representation



Tessellation

Grid-Snapping

- Solution: threshold the distances between the created vertices and the cube corners
- When the distance is smaller than d_{snap} we snap the vertex to the cube corner
- If more than one vertex of a triangle is snapped to the same point, we discard that triangle altogether



Tessellation

Grid-Snapping

- With Grid-Snapping one can obtain significant reduction of space consumption

Parameter	0	0,1	0,2	0,3	0,4	0,46	0,49 5
Vertices	1446	1398	1254	1182	1074	830	830
Reduction	0	3,3	13,3	18,3	25,7	42,6	42,6

Tessellation

Sharp corners and sharp edges

- (Kobbelt et al. 2001):
  - Evaluate the normals
  - When they significantly differ, create additional vertex